Macroeconomic forecasting and analysis at the Czech National Bank: a review

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31 July 2024

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Preface and acknowledgements

Since January 1998 Česká Národní Banka has conducted its monetary policy within an inflation targeting framework. In the inflation targeting framework, under normal circumstances, the Board of the CNB uses a short-term nominal interest rate (the 2W repo rate) as an instrument to achieve its objective—price stability in the form of a publicly announced growth rate of consumer prices over a specified period of time.

Over the years the staff of the CNB's Monetary Department have developed a number of tools and processes to provide analytical support to the deliberations of the Board. In the spring of 2024 I accepted an invitation by the Governor Aleš Michl and the Board to conduct an external review of the models and processes. My contribution is a part of a broader assessment consisting of three external reviews. The reviews are independent of each other and differ in their remits. It is the first assessment of this kind conducted at the CNB, at least since the adoption of inflation targeting.

The original remit of my contribution, as announced by the Governor at the Discussion Forum at the University of Pardubice on 23 April 2024, was to focus only on the models used for macroeconomic forecasting and analysis. However, it became clear early on that it would make little sense to review the models in isolation, without paying regard to the processes in which they are used. The original remit of my review was therefore extended to cover both the models and processes involved in the forecast and briefing rounds in the Monetary Department.

My remit does *not* include the assessment of the inflation targeting framework itself, the inflation target, past monetary policy decisions, or the assessment of the models and processes used at the CNB outside of the Monetary Department. The terms of reference also do not include the assessment of the communication of the Board and the CNB with the public (such as the use of fan charts and scenario analysis in official publications). The only apparent exception occurs when the documents available at some point to the public coincide or overlap with internal briefing materials. The focus is, however, on the documents as a part of the internal debate.

In the process of conducting the review I spent two weeks at the CNB and held in-person individual meetings with all seven members of the Board (typically in the presence of their advisors), a group meeting with the Board advisors, and over a dozen of meetings with the members of the Monetary Department (and the Director of Financial Research Division). All members of the Board and the staff were incredibly generous with their time, helping me understand the workings and challenges of the internal forecast and briefing processes. The members of the forecast team were very patient in trying to help me grasp the details of the models and forecasts. I would especially like to express my gratitude to the advisor to the Governor Tomáš Adam and Stanislav Tvrz, František Brázdik, Karel Musil, Jakub Matějů and Petr Král from the Monetary Department.

To gain insights into the forecast and briefing processes, I attended selected meetings of the Monetary Department, including two meetings with the Board and a meeting with financial market analysts, in the run-up to the interest rate setting decision on 2 May 2024 and the publication of the Spring 2024 Monetary Policy Report. During the process I had access to internal briefing materials.

To benchmark the main macroeconomic model of the CNB, I reviewed models at other central banks on the basis of their public documents. In addition, to gain deeper insights into the experience with the models and briefing processes (and the resources supporting them) at other central banks, I reached out to personal contacts amongst senior staff at the respective banks and former board/interest rate setting committee members. The informal interviews, held in total with 19 people, cover an eclectic sample of 16 institutions, including six central banks conducing independent monetary policies: the Bank of England, Federal Reserve Board, Danmarks Nationalbank, Norges Bank, Sveriges Riksbank, and the Reserve Bank of Australia; five of which, like the CNB, operate under an inflation targeting regime. Three other central banks in the sample are part of the eurozone (the Deutsche Bundesbank, Banque de France, Banco de Portugal), five are regional banks of the Federal Reserve System (Atlanta, Minneapolis, New York, San Francisco, and St. Louis) and two are respected independent research and forecast institutes, the National Institute of Economic and Social Research and the Qatar Centre for Global Banking & Finance, based in the United Kingdom. I am grateful to all the participants for their time. I also drew on my own experience at the Bank of England and regular visits to the Federal Reserve Bank of St. Louis.

The review tries to provide an objective assessment and a constructive feedback on the models and processes supporting monetary policy deliberations at the CNB, drawing not only on the current and past experience of the CNB but also on academic literature and the lessons from other central banks.

The review is divided into four parts. Part 1 starts with conceptual remarks about some important aspects of macroeconomic forecasting and analysis at central banks. To benchmark CNB against its peers, Part 2 provides an overview of the models used for macroeconomic forecasting and analysis at other central banks, pointing out their similarities and differences, as well as advantages and disadvantages. Part 3 focuses on macroeconomic models and their use and scope at the CNB. The recommendations follow in Part 4. Some recommendations can be implemented over a relatively short horizon (within one to two years), while others will require more time (up to three to four years).

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31 July 2024

Executive summary

Since the adoption of inflation targeting, the CNB has developed a sophisticated forecasting and briefing system, the FPAS, in which the main macroeconomic model, the g3+ model, plays a central role. After reviewing the models and the processes, and comparing them with the available alternatives and practices at other central banks, I came to a number of recommendations, which will hopefully further improve macroeconomic forecasting and analysis at the CNB. Some recommendations can be implemented over a relatively short horizon (within one to two years), while others will require more time (up to three to four years). Some recommendations are quite concrete, while others provide more general guidance. Some of the longer-term recommendations can only be implemented in conjunction with each other, provided sufficient resources are made available to support them. A comprehensive discussion of the recommendations, including suggested time frames, is contained in Part 4 of the review.

Recommendation 1. The CNB should expand its analytical toolkit to sufficiently manage the uncertainty around its projections. A number of adjustments should be made to g3+ and to the way the model is developed, maintained and used, as specified in some of the separate recommendations that follow.

The extent, scope and use of various types of models vary across central banks. The CNB appears to be at one end of the spectrum, with g3+ playing a more central role than similar models at other central banks. It would seem prudent to complement g3+ with additional models to address model uncertainty, which necessarily surrounds any forecast. Some of the longer-term suggestions in this direction are covered by my other recommendations. In the short term, g3+ could be complemented with various time series models, which are relatively low-cost to develop and maintain.

Recommendation 2. As a part of forecast risk assessment, there should be more scope in the forecast process for challenge to the projections and the accompanying narrative.

The forecast should remain firmly in the ownership of the staff. But that does not mean it should be presented to the Board as the staff's consensus forecast. Currently, the only formal challenge to the forecast comes in a written form at the end of the process from advisors to the Board and the Financial Stability Department. My recommendation is to bring the challenge forward, change its format and involve also the Monetary Department's staff, with the aim of promoting a livelier discussion of the current conjuncture and risks to the projections than in the current setup.

Recommendation 3. A sustainable long-term process should be put in place for modifications of g3+ and its satellites, to capture alternative transmission mechanisms and to address new policy challenges.

All economic models, however good, represent simplification of reality. Over time, main macroeconomic models at central banks tend to get larger and hard to understand. The CNB should establish a sustainable long-term process, possibly based on smaller models, that is going to guide the staff and the Board in terms of what potential features and transmission mechanisms are worth explicitly incorporating into g3+ and which can be safely left out.

Recommendation 4. Some of the conjunctural and due diligence analysis during forecast rounds could be deemphasized and more resources allocated to addressing long-term questions related to monetary policy strategy and risks to the economy not captured by g3+.

Small dedicated models are often used at central banks to address high-level monetary policy questions and analyze risks not captured by the main macroeconomic model. While such work streams may not address questions of individual forecast rounds, they help policymakers formulate the intellectual basis for the overall monetary policy strategy and its robustness.

Recommendation 5. During forecast rounds, the shocks (residuals) in g3+ could be used as a guidance for further analysis and for building a narrative around the projections.

One of the advantages of g3+ is that such models go beyond partial causal relationships between macroeconomic variables and facilitate a decomposition of the observed data in terms of residuals with economic interpretation. The residuals pick up economic forces around which the forecast narrative can be built, ideally supported by additional evidence.

Recommendation 6. The normative interpretation of the current monetary policy rule in g3+ needs to be deemphasized.

The policy rule in g3+ is typically interpreted as a rule ensuring the achievement of the CNB's inflation target at the policy horizon. This outcome depends, however, also on other parameters of the model, unrelated to monetary policy. The rule simply provides the agents in the model with guidance for forming expectations about monetary policy. The policy rule can also be specified to respond to variables other than (expected) inflation and not violate the inflation targeting objective.

Recommendation 7. The UIP component of the transmission mechanism should be deemphasized and possibly removed.

The UIP systematically fails in the data. It would be advisable to deemphasize the UIP in the exchange rate determination in g3+ and possibly model the exchange rate as a random walk process. Work streams based on high-frequency identification strategies could be used to provide any additional insights into the exchange rate transmission mechanism.

Recommendation 8. In periods of extreme events, it needs to recognized that the structural parameters of g3+ are not strictly invariant. In such circumstances, the relevant parameters should be recalibrated.

The parameters controlling the willingness of firms to change prices, for instance, are not invariant outside of g3+ and, therefore, can be instable in periods of high inflation. Their instability has implications for, among other things, the model's projections for inflation.

Recommendation 9. The documentation of g3+ and its satellites needs to be improved and non-critical features removed from the model.

All relationships within the g3+ model, and between the model and its satellites, need to be more precisely documented in a mathematical form. The parameterization methodology and the parameter values need to be clearly described. The staff should also take stock of the shocks in the model and reassess the way the fiscal impulse is modeled.

Recommendation 10. The presentation of the forecast should clearly indicate which projections come from g3+ and which come from its satellites or near-term-forecast models.

The CNB forecast is an integrated forecast involving g3+, its satellites, near-term forecast models, and judgement. The internal briefing documents would be more transparent, and thus more open to internal scrutiny and challenge, if each chart clearly indicated what models the projections in the chart are based on.

Recommendation 11. The CNB needs to ensure it keeps the technical know-how and human capital to operate, maintain and develop its analytical tools.

The human resources strategy needs to recognize that the maintenance and development of g3+ relies on a very small group of highly technically skilled staff members.

Recommendation 12. The CNB needs to invest in economic research to be better prepared for new challenges, to keep up-to-date with the relevant techniques and literatures and maintain healthy internal exchange of alternative views and ideas.

Forecast rounds at central banks operate under tight schedules. In such an environment, it is easy to stop seeing the forest for the trees, to lose contact with advances in research and for groupthink to develop. The CNB should follow other central banks and invest substantially more in economic research. Not only by providing sufficient resources to the existing research units, but also by having a long-term plan for expanding its research capacity.

Recommendation 13. The CNB should continue investing in its databases. However, some areas of its data management need to be reorganized to free up the time of the analytical staff.

The CNB has acquired a number of micro-level databases and engaged in webscraping. Most of the actual data management should be off-loaded from the Monetary Department to a data management unit to ensure the staff have sufficient time to analyze the data.

1. Macroeconomic forecasting at central banks

To put the review into a broader context, let me start with conceptual remarks about some important aspects of macroeconomic forecasting and analysis specific to central banks. These remarks set the ground for some of the concrete recommendations.[1]

(a) Forecasting at central banks vs. in the private sector. Forecasting the macro economy is not an easy task. Forecasts are subject to unpredictable events (eg, an outbreak of a pandemic or a war) as well as uncertainty about the underlying models used to produce the forecasts and the values of the model parameters. Forecasting at central banks, however, involves an additional layer of complexity. Central banks, unlike private and commercial forecasters, are not just passive takers of macroeconomic developments, they play an active role in shaping them, which stems from their monopoly power over the economy's unit of account. Consequently, central bank forecasts need to take into account how the central bank's own decisions will affect the economy.

(b) The need for a structural view. To meet the inflation target, it is not sufficient for the central bank to come up with a good forecast for the macro economy. The policymakers need to understand also the channels through which their instrument, which under normal circumstances is a short-term nominal interest rate (such as the 2W repo rate), affects the broader economy. For this reason, central bankers need to take a *structural view* of the economy, which spells out the assumed workings of the economic system, as opposed to a *statistical view*, which simply provides a forecast on the basis of the statistical relationships in the historical data.

(c) Private sector expectations. If the economy was a mechanical system of hard-wired relations among economic variables, then the optimal decisions by the central bank—with the objective of meeting its inflation target—would be relatively straightforward. Having a good understanding of how the system works, the central bank would simply need to 'back out' the level of the short-term nominal interest rate consistent with the inflation target. This approach to meeting the objectives of monetary policy dominated the central banking paradigm in the 1960s-70s, as reflected in the use of the large-scale macroeconomic models in the tradition of the Cowles Commission and the use of optimal control borrowed from engineering.[2] The same approach could also be used if economic agents (households, firms, financial market participants) did not try to second-guess what the central bank may do. That is, if their own decisions were immune to the yet-unknown future decisions of the central bank.

(d) Conditional vs. unconditional forecasts. If economic agents try to second guess the future decisions of the central bank, the structural view needs to incorporate assumptions about the expectation formation of the private sector, including expectations about the central bank's behavior. In this context, forecasters distinguish between conditional and

[1] In this introductory discussion I put equality between the central bank and monetary policymakers—at this stage it is not necessary to distinguish between the staff (the Monetary Department in the case of the CNB) and the policymakers (the Board in the case of the CNB).

[2] See, eg, Evans (1969).

unconditional forecasts. Under a *conditional forecast*, the forecast by the central bank is conditioned on a specific path of future short-term nominal interest rates that the bank uses as its instrument. Typically, a constant path or a path implied by futures markets is assumed. Under an *unconditional forecast*, the structural model used by the central bank specifies the central bank's own *reaction function*—an equation describing how the policymakers would respond to changes in various macroeconomic variables. The CNB follows the latter approach. Under both conditional and unconditional forecasts, further assumptions are made about the extent to which the private sector knows the assumed behavior of the central bank.[3] Nonetheless, the term 'unconditional forecast' is somewhat misleading, as clearly the forecast is conditional on the assumed reaction function of the central bank and the assumptions about expectations formation by the private sector.

(e) Internally consistent forecast of interest rates. Regardless of whether the forecast is conditional or unconditional, it is subject to the following problem. Whenever the decision of monetary policymakers deviates from the path of interest rates in the forecast, it potentially invalidates the entire forecast, including the projections for inflation and GDP. This problem is straightforward to understand in the case of a conditional forecast. To validate the forecast, monetary policy decisions would have to follow either the constant path of interest rates or the path implied by futures markets, depending on the running assumption. In the case of an unconditional forecast, the problem is slightly more subtle. As noted in the preceding paragraph, in this case the structural model contains the central bank's own reaction function. The forecast for interest rates that comes out of the model is thus *internally consistent*, as it is generated together with all other macroeconomic variables under a specification that takes into account the responses of the central bank to the macro economy, both now and in the future.

(f) Policymakers' deviations from the internally consistent forecast. As any model is necessarily a simplification of reality, both in terms of its structure and the variables that enter it, so is the reaction function of the central bank in the model. Thus, unless in reality monetary policymakers behave in the (necessarily simplified) way assumed by the model, their decisions are likely to deviate from the model consistent path of interest rates. This subtle point has important implications for the interpretation of the reaction function in the forecasting model, the ownership of the forecast within the institution (ownership by the staff vs. the policymakers), and the models and processes used.[4]

[3] Under the so-called 'rational expectations' the private sector knows the exact path of interest rates under a conditional forecast and the exact form of the reaction function, including its parameter values, under an unconditional forecast.

[4] There are also important implications for the communication with the public, but these are not discussed in this review, as per the review's remit.

2. Models used at central banks

To compare macroeconomic modeling at the CNB with the spectrum of the available options, this section provides an overview of the models used at other central banks for macroeconomic forecasting and analysis, discussing their advantages and disadvantages. The main macroeconomic model of the CNB, the (facelifted) g3+ model, belongs in the class of the so-called DSGE models. I therefore go into some level of detail when discussing DSGE models, to which I refer back in both the discussion of g3+ and the recommendations.

(a) Available options. What are the options when it comes to macroeconomic modeling? Macroeconomic forecasting and analysis at central banks typically involve the horizon of up to three years[5] and there are three main categories of models used for this purpose.[6] The three categories come under the labels *time series*, *semi-structural*, and *structural* models. The last category is also frequently referred to as *dynamic stochastic general equilibrium (DSGE)* models. The narrow use of the acronym DSGE is, however, somewhat misleading, as all three aforementioned categories are DS. That is, the models are dynamic and stochastic, reflecting the fact that the world has a time dimension and is subject to random events.[7] What differentiates the three classes of models is the extent to which their construction is driven by economic theory vs. the historical properties of the data. The boundaries are, however, getting blurred.

Time series models

(b) Time series models. Time series models are statistical models in which the current values of a set of variables are explained by the past values of the same variables. The models are theory free (economic theory may, for instance, dictate only the variables to be included) and therefore flexible in terms of the fit to the data. They are relatively simple to implement in practice, as many commercial and freeware econometric packages include them as a standard feature. Examples of time series models are Box-Jenkins ARMA models, dynamic factor models and vector autoregressive (VAR) models. The last category includes variants such as time-varying VARs, Bayesian VARs (BVARs), smooth transition VARs, threshold VARs, and time-varying volatility VARs (the latter three having the ability to pick up various nonlinearities and regime switches in the data).[8] According to European Central Bank (2021) and Benecká, Kábrt, and Komárek (2024) time series models are rarely used as the main tool for macroeconomic forecasting and analysis at central banks. Their finding has been confirmed in the interviews with my contacts (the exceptions are some of the regional banks of the Federal Reserve System). The main objection against the use of time series models as the main modeling tool is that their purely statistical nature does not allow an interpretation of the forecast in terms of a structural narrative. In addition, time series

[5] The forecast horizon of the CNB is six quarters.

[6] A separate class of models is used for nowcasting—a process of using various high-frequency indicators to estimate the current state of the economy ahead of official data releases.

[7] In all three types of models, macroeconomic variables are mutually related across different time periods and are subject to unpredictable shocks.

[8] Another variant, structural VARs (SVARs), impose restrictions on the model structures, taken from economic theory, to allow a decomposition of the data in terms of shocks with economic interpretation.

models do not allow scenario analysis and policy experiments.[9] Nonetheless, time series models play a role in short-term forecasts and as a complementary source of information for the main projections. Their value is in letting the 'data speak'. For instance, the ECB and the Bank of Canada maintain time series models as robustness and risk management tools for their main economic projections (European Central Bank, 2021; Gosselin and Kozicki, 2023; European Central Bank, 2024). Such use of time series models has been confirmed by a number of my contacts at other central banks.

Semi-structural models

(c) Semi-structural models: 1st generation Semi-structural models are the longest-serving tool for macroeconomic forecasting and analysis at central banks, going back to the 1960s-70s. However, they have undergone significant conceptual revisions since then. The original semi-structural models were based on the Cowles Commission methodology of building large-scale models of the economy loosely based on a combination of both neoclassical (permanent income hypothesis, production function) and Keynesian (hand-to-mouth consumers, price rigidities) paradigms. Private sector expectations were largely absent from the models. The original model of the Federal Reserve Borad (the MPS FRB-MIT-Penn model) is a prime example of this type of models.[10]

(d) Semi-structural models: 2nd generation The current generation of semi-structural models reflects the critique of the original models from the advances in both macroeconomic theory (rational expectations and the Lucas critique) and econometrics (VAR models, non-stationarity and cointegration, and the general to specific approach to dynamic econometric modeling).[11] The first semi-structural model to undergo a major revision was the MPS model of the Federal Reserve Board in the mid-1990s. Relatively recently this revision had a large impact on some other central banks, as discussed in paragraph 2(g).

(e) Semi-structural models: the revamped FRB/US model.[12] The revised version of the semi-structural model of the Federal Reserve Board, the FRB/US model, shares with its predecessor (i) a detailed modeling of the different sectors of the economy (for instance, it includes separate equations for consumption of nondurables and services, automobiles, and other durables as an integral part of the model), (ii) the property that in the long run output is determined by a neoclassical production function, whereas in the short run output is determined by the demand for the different components of the expenditure side of GDP,

[9] Another limitation is that, as the models rely only on the past statistical relationship among the variables, they are unsuitable for situations in which the economy faces new types of shocks (see, eg, Bobeica and Hartwig, 2023, for the performance of VAR models during the COVID-19 pandemic). However, in periods when the economy is not experiencing unusual shocks and monetary policy has not changed its reaction function, VARs capture the average effect of monetary policy on the economy, as well as the endogenous responses of monetary policy to the economy.

[10] Other prominent examples are the Brookings model, the Wharton model, the Michigan model, the BEA model, the DRI model, and their predecessor the Klein-Goldberg model; see, for instance, Duesenberry, Fromm, Klein, and Kuh (1969), Evans (1969), and Fair (1984).

[11] See, eg, Favero (2001) for the exact nature of the critiques.

[12] This paragraph draws on Federal Reserve Board (1996) and Brayton, Levin, Tryon, and Williams (1997).

and (iii) the individual equations are specified and estimated in isolation from each other, without a guarantee they are mutually consistent. The revamped model reflects, however, sounder microeconomic foundations and includes explicit modeling of expectations. Thus, as in the case of DSGE models, economic theory enters more strongly into the current FRB/US model than into its predecessor. The structure of FRB/US consists of households, firms and financial markets sectors, with clear mapping into national accounts.[13] The decisions of households and firms are based on dynamic optimization in the presence of polynomial adjustment costs. The resulting equation for a given variable in this framework depends on a target, dictated by an optimality condition in the absence of adjustment costs (eg, consumption is related to permanent income), and past and expected future values of the variable, reflecting a gradual convergence to the target due to the adjustment costs.[14] The number of leads and lags, and their weights in the equation, are dictated by the empirical fit of the equation. Expectations can be formed on the basis of a small VAR (specified and estimated independently), reflecting bounded rationality, or can be fully rational. The equation for the optimal target is estimated from a long-run relationship between the variables in the equation. Monetary policy is modeled as a part of the VAR, in which the first equation is interpreted as a reaction function for the short-term nominal interest rate. Financial markets are modeled via no-arbitrage conditions (with exogenous risk premia) connecting the short rate to long rates and the stock market capitalization. Simple regressions then connect market rates to mortgage rates, auto loan rates and commercial paper rates, which affect household and firm decisions. The monetary transmission mechanism works mainly through short- and long-term interest rates affecting different components of consumption and investment (through price and wealth effects), while ad-hoc extensions allow for cash flow effects, capturing the presence of hand-to-mouth consumers and financially constrained firms. Prices (inflation) adjust sluggishly to the optimal target of price-setting firms. Money is not an integral part of the model, although an extension to deal with quantitative easing has been developed.[15]

(f) Semi-structural models: the Bundesbank macroeconometric model. While the FRB/US model incorporates responses to both the theoretical and econometric critiques of the first generation of semi-structural models, other semi-structural models used at central banks do not go that far. The macroeconometric model of the Bundesbank is a good example (the model is descibed in detail by Haertel, Hamburg, and Kusin, 2022). Its structure is similar to that of FRB/US[16] but rather than relying on dynamic optimization in the presence of polynomial adjustment costs, the short-run dynamics of the equations pertaining to households and firms are specified and estimated as an error correction model. This specification puts the model on more solid econometric foundations relative to the first

[13]There are also exogenous government and foreign sectors.

[14]Except for the presence of the forward-looking part, from the standpoint of econometric analysis, the resulting equation has the form of a partial adjustment model used in dynamic econometrics.

[15]In the first generation of the semi-structural models, nominal monetary quantities affected short-term interest rates as in the LM curve of the IS-LM model. Real monetary quantities generated quantitatively small wealth effects in the household budget constraint (Duesenberry et al., 1969; Brayton et al., 1997). The switch in the revamped model to model the process for the short-term nominal interest rate directly (through the VAR) rendered the monetary block redundant.

[16]There are households and firms sectors, financial markets, a fiscal sector, and an external sector. The monetary transmission mechanism is also similar to that in FRB/US.

generation of semi-structural models. The resulting equations for households and firms look similar to those of FRB/US, except that the short-run dynamics do not contain the forward-looking (expectations) terms and the long-run relationships are based on loose theoretical arguments. These choices are made on practical grounds to simplify the model specification, estimation, and solution. The obvious limitation is the inability to carry out experiments affecting expectations.

(g) Semi-structural models: use and scope at central banks. Semi-structural models are still widely used at central banks. According to European Central Bank (2021), semi-structural models are used for the main projection at central banks in the following countries (monetary unions): the United States, Canada, Belgium, Germany, Ireland, Greece, Spain, France, Italy, Cyprus, Lithuania, Luxembourg, Malta, the Netherlands, Austria, Portugal, Slovenia, Slovakia, and the ECB. A semi-structural model (the well-known NiGEM) is also used by NIESR. Such widespread use is also confirmed by Benecká et al. (2024). It may be that the reason for the extensive use is simply inertia. Nonetheless, the ECB made a conscious decision to develop a new semi-structural model (ECB-BASE), based on FRB/US, to complement its existing DSGE model. The Banque de France, after an external review, switched from a first generation semi-structural model to a new model (FR-BDF) that closely resembles FRB/US. The Bank of Canada has also built a FRB/US-style model (LENS). Some institutions use semi-structural models not only for projections but also for scenario analysis and policy experiments. In terms of projections, in most cases, the semi-structural model is not the only input into the projection. Other models and staff judgement are also used. This follows from European Central Bank (2021) and Benecká et al. (2024), as well as from the interviews with my contacts.

(h) Semi-structural models: comparison with time series models. The difference between the two classes is obvious: the lack of economic structure in the case of time series models. But there are also similarities to be aware of, which somewhat undermine the structural nature of semi-structural models. First, although some of the short-run dynamics in semi-structural models are motivated by adjustment costs, ultimately the number of lags (and leads) and their weights in the equations are determined by the fit to the data. Thus, as in the case of time series models, a regression, rather than economic theory, does most of the heavy lifting for the short-run behavior of the model. Second, in both cases, the shocks to the model are residuals appended to the equations, picking up movements in the data unexplained by the model equations. Consequently, the economic interpretation of the shocks in semi-structural models is dubious.

(i) Semi-structural models: advantages. Semi-structural models are praised by policymakers for their flexibility (eg, Constâncio, 2017). This virtue was also mentioned to me by a former member of the FOMC, who appreciated the flexibility of the FRB/US model to incorporate aspects of the economy relevant for quantitative easing in a relatively short period of time. What is meant by flexibility is the practice of adding new variables into existing equations or entering new blocks of equations into the model (or removing existing blocks from the model), without the need to adjust the rest of the model. What allows this practice is the fact that the equations of the model are specified and estimated in isolation, independently from each other.[17]

[17]An example of adding new variables into model equations is the addition of various cash flow proxies

(j) Semi-structural models: disadvantages. Flexibility also involves risks. For instance, it may seem natural to include a proxy for mortgage payments into the consumption equation to capture the negative effect of higher mortgage payments on the disposable income of mortgagors. But, the opposite effect on savers, who implicitly finance mortgages, may be ignored. Apart from internal inconsistencies, the other limitations of semi-structural models, as I see them, are as follows: (i) The models are too detached from academic literature, which creates the practical problem that the technical expertise lies with the Federal Reserve Board. Indeed, the new semi-structural models of the ECB, the Banque de France and the Bank of Canada are clones of FRB/US.[18] (ii) Operating semi-structural models, regardless of whether FRB/US-style or not, requires specialized knowledge, which can be acquired only by learning-by-doing. As the equations are specified and estimated in isolation from each other, there is no guarantee that the model as a whole will perform well (or even converge). My contacts among those who operate such models pointed out that interventions are often needed to adjust the estimated parameters to ensure an acceptable behavior of the model. (iii) Even though the equations are estimated individually to maximize their fit, the R^2 can be quite low, as in the case of consumption (eg, the FR-BDF model; see Banque de France, 2019). (iv) Due to the complexity of the models, issues related to the estimation of the parameters, such as identification, endogeneity and cointegration, are often swept under the carpet, raising questions about the validity and interpretation of the estimated parameters. (v) While the basic structure of semi-structural models is quite straightforward, the dynamic interactions among the variables are not easy to understand. Finally, (vi) issues related to the short-run dynamics and the interpretation of the shocks have been raised in paragraph 2(h).

DSGE models

(k) DSGE models: use and scope at central banks. Since their inception in the early 1980s, (quantitative) DSGE models have become the main paradigm in macroeconomics. Nowadays they are used to address a wide spectrum of questions, from monetary policy to health and wealth inequality. New parameterization and solution methods have developed in lockstep. Some central banks adopted DSGE models, as the main forecasting and analytical tool, after three influential papers showed that a specific type of this class of models, the so-called new-Keynesian (NK) models, when appropriately extended, fit well the key US and eurozone macroeconomic data and can replicate the responses of the economy to unexpected changes in the policy interest rate (Smets and Wouters, 2003; Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007).[19] DSGE models at large, and DSGE-NK models in particular, came under heavy criticism during the global fi-

into the short-run dynamics of consumption and investment, to capture the presence of credit constrained households and firms (and to improve the fit of the equations). Such additions are apparent both in the FRB/US model and the new semi-structural model of the Banque de France. An example of adding a whole new block is the addition of quantitative easing to FRB/US.

[18] The simpler models among the second generation of semi-structural models, the models based on error correction econometrics, did not pass earlier external reviews at the Banque de France and the Bank of England, leading to the development of FR-BDF and BEQM (later COMPASS), respectively.

[19] Simpler variants had been already used at central banks as an organizing framework for policy analysis; see, eg, Bank of England (1999), Chapter 4.

nancial crisis of 2008 and the following Great Recession, as the models we unable to address key policy questions. In response, some central banks have developed semi-structural models (see paragraph 2(g)), while others have extended their DSGE-NK models by incorporating banking and housing sectors.[20] According to European Central Bank (2021) a DSGE-NK model serves as the main forecasting tool at central banks in the following countries: Sweden (Ramses, MAJA), the United Kingdom (COMPASS), Latvia, and Finland (Aino). In addition, a DSGE-NK model is used as the main forecasting tool at the Norges Bank (NEMO), which is not listed in the ECB publication, and there is an on-going effort at the Danmarks Nationalbank to develop such a model. Among the regional banks of the Federal Reserve System, the New York Fed, for instance, maintains a DSGE-NK model for forecasting. Some of the central banks surveyed by European Central Bank (2021) use a DSGE-NK model in the forecasting process in conjunction with a semi-structural model and all but one of the institutions use DSGE models for policy analysis. These findings correlate with the findings of Benecká et al. (2024). According to my contacts, when a DSGE model is used as the main forecasting tool, it is rarely used on its own. Apart from judgement, time series models are used as a cross-check and risk management tool. Various additional DSGE models are also used to address specific ever-present, strategic, or ad-hoc questions. To mention just a few such examples, the Bank of England has built an OLG model to estimate the impact of population ageing on the natural rate of interest, the Banque de France uses small DSGE-NK models to study different policy rules, and the Atlanta Fed, shortly after the financial crises of 2008, developed a small DSGE model with financial frictions to better understand the likely speed of recovery of the US economy from the crisis.

(l) DSGE models: strengths, weaknesses, frustrations. As noted in paragraph 2(a), all models used at central banks for macroeconomic forecasting and analysis are DS. The strength of DSGE models is the GE part of the acronym, which imposes internal consistency on the model through the discipline of general equilibrium.[21] The internal consistency is, however, also the main weakness of DSGE models. For most people, a natural way to think about the economy is in a partial equilibrium setting; ie, thinking about the behavior of a particular set of agents or a market in isolation and applying simple causality. A natural reasoning, for instance, is that GDP growth slows down because households spend less and households spend less because wages are growing at a slower pace. In general equilibrium, however, the reverse causality is equally valid. This is because in general equilibrium all three variables are determined simultaneously and are ultimately driven by the state of the economy (eg, the existing infrastructure) and exogenous factors, some of which may be observable (eg, international gas prices), while others may not be directly observable (eg, an increase in uncertainty). The interpretation of a central bank reaction function adhering to the so-called Taylor principle is another example. Based on that equation alone,

[20] The academic DSGE literature responded (mainly) by paying more attention, in a great level of detail, to household heterogeneity and mortgage and housing markets. In contrast, during the COVID-19 and the following high inflation period, central banks, more then the general DSGE paradigm, came under criticism and scrutiny. Focusing on the models, Del Negro, Dogra, Gleich, Gundam, Lee, Nallamotu, and Pacula (2024) compare the performance of the New York Fed DSGE Model—a prototypical DSGE-NK model—before and after COVID-19. Since 2011, the DSGE's accuracy was comparable to that of private forecasters before COVID-19 but worse thereafter.

[21] General equilibrium does not necessarily mean that all markets clear at all times. General equilibrium search and matching models are examples of models in which markets do not clear.

partial equilibrium reasoning suggests that a central bank following the Taylor principle should move the policy rate by more than one-for-one with inflation. However, in many NK settings, the Taylor principle is simply a threat, which if credible, is never carried out in equilibrium and both the policy rate and inflation are perfectly stable (this is similar to the effect of off-equilibrium strategies in game theory).[22] As a final example, most shocks in a general equilibrium setting do not have a simple classification as aggregate supply or demand shocks. A shock to productivity, for instance, affects both production as well as demand for consumption and investment goods. The counterintuitive properties of DSGE models place high requirements on the communication skills of the staff in their interactions with policymakers. From the interviews with my contacts, and my own experience at the Bank of England, it appears that such communication challenges are fairly universal across central banks.

(m) DSGE-NK models: the bare bones, RANK, TANK, HANK. Essentially all dynamic models in economics and finance in which resources can be moved across time (through investment/saving in financial, productive, housing or human capital) contain the following equation (or a set of equations)

$$1 = E_t[m_{t+1}R_{t+1}].$$

Here, m_{t+1} is the so-called stochastic discount factor, R_{t+1} is a gross real (ie, inflation adjusted) rate of return on an asset and E_t denotes the fact that the variables in the parenthesis are unknown in today's period t and a forecast (expectation) has to be formed about their future values. In finance such conditions are called 'no-arbitrage conditions' and m_{t+1} is modeled by sophisticated statistical methods giving rise to various 'factor models'. In economics the equations are called 'Euler equations' and, through optimization by economic agents, m_{t+1} is related to economic fundamentals. Depending on the model, different fundamentals enter m_{t+1}, consumption growth between t and $t+1$ and financial constraints being the most common factors. It is important to be aware that the expectations symbol in the above equation does not imply rational expectations.[23] When the above equation pertains to a one-period nominal bond, it takes the form

$$1 = E_t[m_{t+1}(1 + i_t)/(1 + \pi_{t+1})],$$

where i_t is the short-term nominal interest rate observed in period t and π_{t+1} is the yet unknown inflation rate between periods t and $t+1$.[24] This equation plays a key role in all models in which monetary policy controls the short-term nominal interest rate. In representative agent new-Keynesian (RANK) models, m_{t+1} pertains to the representative consumer. In two-agent new-Keynesian (TANK) models, m_{t+1} pertains to a representative permanent income hypothesis consumer (the other type being a hand-to-mouth consumer).[25] And in

[22] To be precise, the policy rate tracks the natural rate of interest in the so-called 'divine coincidence' case and responds to external shocks in richer settings. However, the threat of the Taylor principle is never carried out. See, eg, Woodford (2003); Cochrane (2011); Galí (2015).

[23] It is not required that the expectations are consistent with the average realized behavior of m_{t+1} and R_{t+1}. Rational expectations, nonetheless, is the most common assumption.

[24] The term 'bond' is used generically to cover any short-term borrowing/saving vehicle.

[25] Other two-agent settings include a borrower-saver, capitalist-worker, and rich-poor settings.

heterogeneous-agent new-Keynesian (HANK) models, which have become popular in the academic literature, m_{t+1} pertains to any agent, in a large cross-section, who is borrowing/saving in the one-period bond. According to the above equation, by controlling i_t, the central bank affects, in expectations, the *product* of m_{t+1} and $(1 + \pi_{t+1})$. Private sector expectations are thus a key part of the transmission mechanism of monetary policy in this setup. There are many potential channels through which m_{t+1} and $(1+\pi_{t+1})$ could be related to each other. In a prototypical DSGE-NK model the connection comes from the so-called NK Phillips curve, according to which individual firms choose to adjust output when they expect a change in aggregate inflation to be only temporary, in order to avoid having to re-set their prices again soon (the underlying assumption is that price changes are costly whereas output changes are costless). As output affects household income, it affects m_{t+1} in the Euler equation. Prototypical DSGE-NK models are thus ultimately models of inflation expectations and price-setting behavior. To the extent that shock persistence affects expectations, it is key for the predictions of DSGE-NK models.

(n) DSGE-NK models: linear implementation and its consequences. In practice, DSGE-NK models are typically 'linearized' (a log-linear approximation is applied to the equations). Consequently, only conditional first moments (expectations) of the model variables remain in the model. For instance, in the above Euler equation, the conditional covariance between m_{t+1} and R_{t+1} disappears from the model. Linearized models therefore remove risk premia and the effects of uncertainty (eg, on precautionary saving) from the model structure, as these concepts depend on higher moments (variances, covariances, fat tails, etc.). While not strictly necessary, linearization is a practical way to solve the large-scale DSGE-NK models used at central banks.

(o) DSGE-NK models: unobservable variables. The linear solution to a DSGE-NK model takes the form

$$y_t = Mx_t, \quad \text{and} \quad x_{t+1} = Ax_t + B\varepsilon_{t+1},$$

where y_t is a set of macroeconomic variables in the current period t, such as current GDP and inflation, x_t is a set of variables describing the current state of the economy, and ε_{t+1} are unforecastable changes in x_t between periods t and $t+1$. The set x_t includes variables that, as of this period, are predetermined (eg, the economy's productive capital stock), past values of the variables in y_t (eg, past short rates if the central bank changes interest rates only gradually), and various factors that are taken as exogenous from the model's perspective (eg, gas prices or the extent of monopoly power in domestic markets). In the above equations, M, A and B are matrices whose elements generally depend on the 'structural' parameters of the model (eg, the parameters of households' and firms' maximization problems), expectations, the persistence of the processes describing the exogenous factors, and the parameters of the monetary policy reaction function.[26] Some of the elements of x_t may be observable (eg, gas prices or past policy rates), some may be estimable with nowcasting methods (eg, the capital

[26]More generally, the above equation for y_t takes the form $y_t = Mx_t + D(z_{t+1}, ..., z_{t+T}) + \xi_t$. Here, ξ_t is a measurement error, capturing the fact that some of the elements of y_t may not be measured precisely in the data (eg, the concept of consumption in the model may not exactly correspond to consumption as defined and measured by the statistical office). $D(z_{t+1}, ..., z_{t+T})$ allows for the possibility that the future values of some variables may be known (eg, if future fiscal plans are known and credible). I abstract here from these complications here.

stock or past values of GDP if yet unreleased by the statistical office), while some may be completely unobservable (eg, the degree of monopoly power). Some of the unobservable variables could be perhaps somehow estimated outside of the model. In practice, however, they are 'backed out' from the model. Loosely speaking, given the observable and nowcastable elements of x_t, the y_t implied by the model is unlikely to fit the data on y_t perfectly. The unobservables pick up this discrepancy so that the model fits the data. The unobservables are thus essentially the model's residuals (or more precisely the residuals of the linear version of the model).[27]

(p) DSGE-NK models: similarities with semi-structural models. The state-of-the art semi-structural models (the FRB/US-style models) and the DSGE-NK models used at central banks are more similar than it may seem. (i) Both types have a clear mapping into national accounts, although the mapping of the semi-structural models is more detailed. The semi-structural models, for instance, include the sub-components of aggregate consumption and investment as integral parts of the model, whereas DSGE-NK models typically work only with aggregate consumption and investment. This is, however, more by the choice of the DSGE-NK modelers at central banks than a generic feature of DSGE models. (ii) Both types of models are also based on a neoclassical production function and dynamic optimization by households and firms, although the details differ. (iii) Money is absent and the transmission mechanism works through interest rates (and exchange rates).[28] Semi-structural models have, however, an explicit role for long-term interest rates and markups in commercial rates. But the absence of such effects in DSGE-NK models is again by the choice of the central bank modelers than a generic property of DSGE models. (iv) Both types of models are implemented in a linear form. Risk premia are exogenous or modeled in an ad-hoc reduced-form way. (v) Both types can accommodate deviations from rational expectations, although rational expectations is the default assumption in DSGE-NK models. (vi) Both types work with data that are seasonally and work day adjusted.

(q) DSGE-NK models: differences from semi-structural models. The main differences are in (I) internal consistency, (II) the role of residuals in short-run dynamics, and (III) the details of the labor market. The advantages and disadvantages of (I) are discussed in paragraphs 2(i), 2(j) and 2(l). In terms of practical implications, the internal consistency of DSGE models means that most of the time it is not possible to add/remove parts of the model without affecting the rest of the model, which makes substantial modifications highly time consuming. To get around this problem, reduced-form quick fixes are often employed (an example is the addition of an ad-hoc risk premium term to a particular equation), a practice bringing DSGE-NK models closer to semi-structural models. Regarding (II), as noted in

[27] In the DSGE literature, the exogenous unobservable variables are referred to as unobservable 'shocks'. The staff at the CNB uses the term 'shocks' for the ε_{t+1} in the above representation of the linear version of the model. I was told the staff adopted their terminology from the IMF. One needs to bear this difference in terminology in mind when reading through the CNB's materials.

[28] For instance, both types of models assume monetary dominance. A situation under which monetary policy is independent of fiscal policy and public debt cannot be monetized. Typically, the sustainability of public debt is guaranteed by the assumption of lump-sum taxes, which are whatever they need to be to make public debt sustainable. The struggle of introducing money into DSGE-NK models is nicely reflected in an ECB conference volume edited by Beyer and Reichlin (2008). Practical problems with the use of monetary aggregates in the conduct of monetary policy, in the case of the UK, are described by Cobham (2003).

paragraph 2(h), the short-run dynamics of semi-structural models are driven by the lags of the model variables, which are determined empirically. In DSGE-NK models, the lags play less of a role, as the model structure is more strongly influenced by economic theory. But even here the boundaries are getting blurred, as DSGE-NK modelers tend to evoke more or less theoretically sound arguments to include more lags (eg, habits in consumption or complex investment adjustment costs). Nonetheless, residuals still do relatively more of the heavy lifting for short-run dynamics in DSGE-NK models than in semi-structural models. In semi-structural models the residuals are literally regression error terms. In DSGE-NK models it is easier to assign an economic interpretation to them, as the residuals are directly linked to the micro-foundations and distortions in various equilibrium conditions of the model. However, caution has to be used in order not to interpret the residuals too literally (Chari, Kehoe, and McGrattan, 2009).[29] Finally, regarding (III), DSGE-NK models typically abstract from unemployment. The standard labor market variables in these models are total hours worked and wages. This is more by the choice of the modelers than a general feature of DSGE models.[30]

[29] Having said that, in practice various ad-hoc residuals are sometimes added to the equations of the linear version of the model. In such cases, any economic interpretation is dubious.

[30] Search and matching models of unemployment, for instance, have been firmly established in the DSGE paradigm since the mid-1990s (Andolfado, 1996).

3. Macroeconomic forecasting and analysis at the CNB

(a) FPAS. Since the introduction of inflation targeting in 1998, the staff in the Monetary Department (MD) of the CNB have developed a set of models and processes to help the Board with their monetary policy deliberations. This system, named the Forecasting and Policy Analysis System (FPAS), centers around a main macroeconomic model, the (facelifted) g3+ model. The g3+ model belongs squarely into the DSGE-NK category. A number of satellite models are built around it to either provide inputs into the model or decompose its output. A battery of near-term forecast (NTF) models are used to estimate the state of the economy in the previous quarter, if official data are not yet available, and to provide a forecast for the current quarter.[31] My familiarity with FPAS is based on publicly available CNB working papers, various other complementary pieces published in Monetary Policy Reports and on the CNB's blog, FPAS presentations prepared by the MD for new Board members and financial market analysts, a document prepared by the MD for this review, and numerous conversations with the staff. I also attended (on-line or in-person) selected meetings of the MD, including two meetings with the Board and a meeting with financial market analysts, in the run-up to the interest rate setting decision on 2 May 2024 (and the consequent publication of the Spring 2024 Monetary Policy Report).

(b) The view of the Board. During my first visit at the CNB I held individual meetings with all members of the Board, the main users of the forecast and the accompanying analysis. The purpose of these meetings was to seek the views of the Board members on the models and processes supporting their deliberations. While I asked all Board members the same set of questions, I generally let the discussion flow to allow individual members to sufficiently express their views. Most of the discussion revolved around g3+ and its role. All members of the Board agreed on the need for a model, both as a benchmark for internal discussions and as a communication device with the public. Nearly all members of the Board felt, however, that the current setup is too unicentric, with the debate shaped almost exclusively by g3+. Most Board members thought that the unicentric setup prevents a broader discussion about the transmission mechanism and the risks to the economy and the forecast (but different members had preference for different additional channels, although there was some overlap). The Board members also raised the concern that the focus on g3+ constraints the MD's flexibility to accommodate the Board's analytical requests, although some members noted that there had been improvements in this respect and acknowledged that the Board also needs to make more effort to appreciate the existing analytical framework. One member of the Board made the point that having the forecasting framework built around a single model has the advantage of continuity of the debate both across forecast rounds and consecutive Boards. The appreciation for the details of g3+ seems to differ across the Board members. While some members felt there was little need to know the details and thought that the model, its output and description were satisfactory, others were frustrated by the model's complexity and the lack of transparency of its inner workings and frequent changes to the

[31] I found the use of the terms backcast, nowcast, and near-term forecast in the publicly available documents, and the documents provided to me by the MD, relatively loose. To fix the terminology for the purposes of this review, I will refer to the estimates for the previous quarter as a 'backcast', to the estimates for the current quarter as a 'nowcast', and to the battery of models delivering the backcast and nowcast as 'NTF models'. I reserve the term 'forecast' for the next and later quarters.

forecast narrative. A number of Board members referred to the model as a 'black box'. The view of the Board on the general DSGE methodology seems to be split. While some members were sceptical about its suitability for macroeconomic forecasting and analysis at central banks, one member was positive and others were broadly neutral.

FPAS models

(c) The g3+ model and a comparison with its peers. The g3+ model belongs squarely into the DSGE-NK category. The general properties of DSGE-NK models discussed in paragraphs 2(l) - 2(q), including their advantages and disadvantages, apply in full to g3+. The g3+ model has served as the main forecasting tool of FPAS since 2019.Q3. It is a modified version of its predecessor, the g3 model, which was in use from 2008.Q3 to 2019.Q2.[32] Recently, the g3+ model has undergone minor modifications, described in the Winter 2024 Monetary Policy Report (the facelifted g3+ model). The guts of the model have remained, however, the same since 2008.[33] The g3+ model is similar to the DSGE-NK models used, as the main forecasting and analytical tools, at the central banks in Sweden (Ramses, MAJA), the United Kingdom (COMPASS), Latvia, Finland (Aino), and Norway (NEMO). It shares with its peers the same underlying structure, based on Smets and Wouters (2003), Christiano et al. (2005), and Smets and Wouters (2007), including the standard ad-hoc features, such as habits in consumption, investment adjustment costs, and partial price and wage indexation, to achieve a better fit to the data. The differences across the models at central banks are only in details: whether the model is RANK or TANK, the granularity of the price tree making up the CPI and other price indexes, whether or not the model includes housing, the way the model deals with the fiscal and foreign sectors, the number and types of exogenous shocks, parameterization, and the details of the monetary policy rule. In terms of the details, the g3+ model is a TANK model, with permanent income hypothesis and hand-to-mouth households, without housing but with a detailed price tree. The policy rule responds only to (expected) inflation.

(d) A snapshot of the monetary transmission mechanism in g3+. The model is complex in its details. But the transmission mechanism is effectively the standard transmission mechanism of DSGE-NK models described briefly in paragraph 2(m). The price-setting block, however, is much richer than in a prototypical DSGE-NK model. The CPI consists of indirect taxes, regulated prices, and net inflation. Indirect taxes and regulated prices are exogenous. The net inflation component of the CPI consists of a markup over a weighted average of the prices of domestic goods, imported energy goods and imported nonenergy goods. The last two depend on exogenous foreign prices and the exchange rate. Finally, prices of domestic goods consist of a markup over domestic marginal costs, which are a

[32] The original g3 model is described by Andrle, Hlédik, Kameník, and Vlček (2009); the new features of g3+ by Brázdik, Hlédik, Humplová, Martonosi, Musil, Ryšánek, Šestořád, Tonner, Tvrz, and Žáček (2020).

[33] Before the g3 model, the CNB relied on the Quarterly Projection Model (QPM), a model in which most variables were expressed in terms of gaps from their 'equilibrium' values. QPM, which in terms of the taxonomy of Part 2 falls somewhere between the error-correction semi-structural and DSGE-NK models, was in use between 2002.Q3 and 2008.Q2. Before QPM, forecasts were based on various time series models and expert judgement. The purpose of QPM, and the subsequent models, was to bring consistency into the forecasts from the different models and MD teams. One of the drawbacks of QPM was the requirement to correctly estimate the unobservable gaps.

weighted average of the costs of capital and labor, adjusted for productivity.[34] In principle, the model enables a purely accounting decomposition of inflation. One could exogenously vary each component of the CPI in the model, including markups, and trace out its effect on inflation. In the model, markups, domestic marginal costs and the exchange rate are endogenous. However, the exchange rate depends on an exogenous risk premium and markups contain exogenous 'markup shocks'.[35] To the extent that the risk premium (rather than the UIP) and the markup shocks do most of the heavy lifting in g3+ for, respectively, the exchange rate and markup dynamics, one could think (at the risk of oversimplification) of net inflation in the model as being determined by exogenous movements in foreign prices, the exchange rate and markups and endogenous movements in nominal domestic marginal costs. It is this last component in the price tree that monetary policy in the model has an effect on, through the standard DSGE-NK mechanism described in paragraph 2(m).[36]

(e) Satellite models. The MD uses a whole battery of satellite models in FPAS. The satellite models span semi-structural, econometric, time series, and high-frequency NTF models. Given their number and diversity, it is impossible to review the models here in much detail. Most of the satellite models are either add-ins or add-ons (these are my own labels). In addition, a few models are neither add-ins nor add-ons and run in parallel with g3+. Add-ins provide inputs into g3+, while add-ons decompose the output from g3+ into variables that are not an integral part of the g3+ model. Add-ons also can be used to carry out additional analysis of the output of the add-ins. Examples of models that run completely in parallel are models for estimating the natural rate of interest and the output gap.

(f) Satellite add-ins. There are three main add-ins, the fiscal block, the external block, and NTF models for backcasting and nowcasting. The fiscal block works roughly as follows: An expert team reviews the government fiscal plans and identifies changes in (i) government consumption, (ii) other expenditures and (iii) taxation (including social contributions etc.). A set of multipliers estimated outside of g3+ (or guided by the literature) are then applied to the various components making up government expenditures and taxation to work out the so-called 'fiscal impulse'; ie, the effect of these measures on GDP, consumption and investment. The fiscal impulse is then required to be replicated by g3+. This is engineered through shocks to household preferences (consumption habits), investment adjustment costs, and government consumption, which enters g3+ directly through the government budget constraint. The external block delivers forecasts for (i) the effective EA real GDP, (ii) the

[34] There is a corresponding production tree in the model. Indeed, the price and production trees are jointly implied by the production functions and the price setting behavior of firms in the model. Long-run differences in productivity growth at the different stages of the production tree generate a 'convergence component' in the price tree. In recent years, this effect on the CPI in the model has been relatively small.

[35] Without an exogenous risk premium DSGE-NK models, like most models, cannot fit exchange rate dynamics, which in the data exhibit significant departures from UIP. Mark-up shocks play an important role in the overall fit of DSGE-NK models. A part of the risk premium in g3+ is endogenous, linked to the net foreign asset position in an ad-hoc way. But this is just a well-known technical condition to guarantee stationarity of the linear version of the model. As a first pass, the risk premium is exogenous.

[36] In addition to NK Phillips curves in prices the model contains also a NK Phillips curve in wages. This feature introduces another layer of price-setting behavior into the price tree (setting of nominal wages by workers) but does not change the basic description of the mechanism. The combination of the price and wage setting components of the price tree, together with inflation expectation formation in the model, can be thought of as the model's wage-price inflation spiral.

effective EA PPI, both energy and nonenergy, and (iii) the nominal EURIBOR 3M interest rate and its shadow unconventional counterpart. The forecasts are based on various time series and econometric models and my understanding is they are also somehow integrated with forecasts of Consensus Economics and Oxford Economics.[37] Finally, the NTF models provide an estimate of the past and current state of the economy. Their precise role is described in paragraph 3(h).

(g) Satellite add-ons. The add-ons are primarily used to obtain forecasts for additional variables of interest that are not an integral part of g3+. The objective is to obtain forecasts for these variables that are consistent with the projections from g3+. The general, though not the only, principle is to exploit statistical relationships between the additional variables of interest and the macro variables that are an integral part of g3+. At this stage, however, the right-hand side of the regressions is based on output from g3+. A number of the add-ons are based on the NTF models for the respective variables (eg, add-ons for various inflation decompositions are based on NTF models for inflation). The output of fiscal add-ons provides a forecast for government revenues, expenditures and deficits. A number of add-ons are used for the external block. A semi-structural (gap) model is applied to the forecast generated by the external block's add-ins to provide its structural interpretation. Further, the NiGEM global model maintained by NIESR is used to model and interpret alternative scenarios for the external sector. Finally, there are add-ons to decompose the g3+ forecast of the balance of payments into various subcomponents not captured by g3+. Outside of the fiscal and external sectors, there are add-ons to decompose the g3+ forecast of investment and inflation into subcomponents not explicitly modeled in g3+ (eg, investment is decomposed into private, public, and a change in inventories and inflation is decomposed into core inflation, food price inflation, etc.). There are also add-ons for the labor market (g3+ works only with total hours and wages), money and credit (g3+ has no money or credit), and house prices (g3+ does not have house prices; the forecast from the house price add-on is used to obtain the imputed rent component in one of the add-on decompositions of the g3+ inflation forecast).

FPAS forecast

(h) The integrated forecast. The CNB forecast is an integrated forecast based on FPAS and owned by the MD. In nutshell (at the risk of oversimplifying), the integrated forecast is produced as follows. Using an extended version of the equation introduced in paragraph 2(o), the g3+ model in its linear form can be characterized as

$$y_t = Mx_t + D(z_{t+1}, ..., z_{t+T}) + \xi_t.$$

The quarter in which the forecast takes place is the quarter $t = 1$. Data for some of the variables in y_1 are already available. For the remaining variables in y_1, nowcast is produced by satellite add-ins (paragraph 3(f)). The variables in x_1 constitute the *initial conditions* of the forecast. They can be either observable or unobservable. Data for some of the observable variables in x_1 are already available in period $t = 1$. For the remaining observable variables, their nowcast or backcast, depending on the variable, are produced by satellite add-ins.

[37] The effective EA consist of Germany, Slovakia, France, Italy, Spain, and Austria, appropriately weighted.

Finally, some variables in x_1 are set by *conditioning assumptions*, which also set their future values $z_2, z_3, ...$. This applies to variables whose current and future values are known (taxes, administered prices) and variables whose forecasts are produced by satellite add-ins (the fiscal and external blocks). Judgement is also implemented via conditioning assumptions.[38] Given all the above inputs, the relationship between the left-hand side and the right-hand side of the above equation is unlikely to hold (paragraph 2(o)). The equality is guaranteed by the unobservable variables in the vector x_1, the *shocks*, whose values are 'backed out' from the equation to ensure that the equation holds. The process of backing out is a little more involved due to the presence of ξ_1 in the above equation, a potential measurement error in the data. This complication is dealt with the use of standard filtering techniques (some data may be pre-filtered before they are used in FPAS). Filtering ensures that various short-term idiosyncratic movements in the data do not affect the overall picture of the economy and the forecast. The evolution of the variables in the state vector x_t follows either the VAR process introduced in paragraph 2(o), $x_{t+1} = Ax_t + B\varepsilon_{t+1}$, or the known paths $z_2, z_3, ...$ In the VAR, ε_{t+1} are unforecastable innovations, pertaining only to the shocks in x_t. Starting with x_1, the forecast for $x_2, x_3, ...$ is generated recursively by the VAR. The forecast for $y_2, y_3, ...$ is then generated by the equation $y_t = Mx_t + D(z_{t+1}, ..., z_{t+T})$. Subsequently, satellite add-ons (paragraph 3(g)) are applied to the forecasts of the y's and x's to generate forecasts of variables not included in g3+. The integrated forecast reflects all the above steps (and possibly various iterations between them).

(i) Forecast performance. The CNB regularly publishes the Assessment of the Fulfilment of the Forecasts as an appendix to its Spring Monetary Policy Reports. In the Assessment, the CNB compares its performance with that of other institutions forecasting the Czech economy. Tables 1 and 2 below, in contrast, compare the forecast performance of the CNB with that of other central banks. The comparison is for one year ahead inflation and GDP growth forecasts, measured by the root mean squared error (RMSE). The RMSEs for the other central banks are taken from Bernanke (2024). The performance of the CNB is comparable to that of the other banks in the sample. In the case of inflation, the RMSE of all central banks significantly worsened during the high inflation period 2021.Q2-23.Q3. During this period, all central banks underestimated the sharp increase in inflation (here, the CNB was somewhat worse than the other banks in the sample). In the case of GDP growth, the RMSE uniformly worsened during the Covid-19 period, 2020.Q1-21.Q1. Unsurprisingly,

Table 1: **RMSEs, one year ahead inflation forecasts**

Period	CNB	BoE	ECB	Riksbank	BoC	Norges Bank	RBNZ
2015-19	0.81	0.64	0.67	0.38	0.41	1.02	0.65
2020.Q1-21.Q1	0.79	1.02	0.96	1.12	1.30	0.71	0.46
2021.Q2-23.Q3	7.53	4.60	4.99	5.01	3.07	3.63	4.22

[38]Deviations of monetary policy from the monetary policy rule in g3+ are also implemented through the $D(z_{t+1}, ..., z_{t+T})$ part. The relevant elements of the D matrix depend on the extent to which the deviations from the policy rule are expected by the private sector (implemented via the LIRE functionality of g3+).

Table 2: **RMSEs, one year ahead forecasts of GDP growth**

Period	CNB	BoE	ECB	Riksbank	Norges Bank	RBNZ
2015-19	1.53	0.85	1.68	1.01	0.76	0.82
2020.Q1-21.Q1	7.79	12.95	7.23	4.44	5.25	5.74
2021.Q2-23.Q3	1.61	3.75	4.99	1.96	0.99	4.05

central banks did not forecast the outbreak of the Covid-19 pandemic and its negative impact on economic activity. However, as is clear from the Assessment in the Spring 2022 Monetary Policy Report, as the pandemic progressed, the forecasting performance of the CNB significantly improved.[39]

[39] During the Covid-19 period, the CNB, like other central banks, departed from the use of its main model and usual processes and placed more emphasis on NTF forecasts and various ad-hoc models.

4. Recommendations

Over the 26 years since the adoption of inflation targeting, the CNB has developed a sophisticated forecasting and briefing system, the FPAS, in which the main macroeconomic model, the g3+ model, plays a central role. After reviewing the models and related processes, and comparing them with the available alternatives and practices at other central banks, I came to the following recommendations, which will hopefully further improve macroeconomic forecasting, analysis and briefing processes at the CNB. Some recommendations can be implemented over a relatively short horizon (within one to two years), while others will require more time (up to three to four years). Some recommendations are quite concrete, while others provide more general guidance. Some of the longer-term recommendations can only be implemented in conjunction with each other, provided sufficient resources are made available to support them.

Recommendation 1. *The CNB should expand its analytical toolkit to sufficiently manage the uncertainty around its projections. A number of adjustments should be made to g3+ and to the way the model is developed, maintained and used, as specified in some of the separate recommendations that follow.*

The g3+ model, a DSGE-NK model, is comparable to the models in this class used, as the main forecasting and analytical tool, at other central banks (paragraph 3(c)). The main potential alternative to g3+ would be a semi-structural model. However, in a number of aspects the state-of-the-art semi-structural models are already quite similar to the DSGE-NK models used at central banks (paragraphs 2(p) and 2(q)). At the same time, it does not appear that one approach dominates the other; there are clear advantages and limitations of each approach (paragraphs 2(i), 2(j), 2(l)). To switch from g3+ to a state-of-the-art semi-structural model, and to master the required techniques to maintain and operate the model, would incur substantial costs. On balance, the advantages and disadvantages of the alternative approach do not seem to justify such a major change in the forecasting and analytical system. Although a switch to a simpler than state-of-the-art semi-structural model (paragraph 2(f)) is more feasible, one needs to bear in mind that such models have not passed some earlier external reviews at central banks (paragraph 2(j)).

At the same time, the CNB should expand its analytical toolkit to sufficiently manage the uncertainty around its projections. The extent, scope and use of DSGE-NK models vary across central banks (paragraph 2(k)). The CNB seems to be at one end of the spectrum, with g3+ playing a more central role than DSGE-NK models at other central banks. It would seem prudent to complement g3+ with additional models to address the model uncertainty, which necessarily surrounds any forecast (currently FPAS addresses uncertainty about the conditioning assumptions and initial conditions, both in the context of g3+). Some of the longer-term suggestions in this direction are discussed in my other recommendations. In the next **one to two years**, the CNB should introduce various time series models into its toolkit. Such practice seems to be common at other central banks (paragraph 2(b)). The CNB already has the technical know-how to build on. The g3+ projections are an integrated forecast, which takes into account the output of a battery of satellite models (paragraphs 3(e)-(g)) as well as judgement. Some of the satellites are based on time series models. Some of these models (eg, NTF models for inflation) already produce forecasts for the entire six-quarter horizon, although only the first one or two quarters are typically integrated into g3+. My recommendation is that, for internal risk management purposes, the staff produce separate forecasts based purely on statistical relationships in the data and present the forecasts to the Board alongside the integrated g3+ projections. It is then for the Board and the MD to debate the extent to which any discrepancies between the integrated forecast and the statistical forecasts should shape the Board's decisions. Deviations of the Board from the interest rate path implied by the integrated forecast are well justified in the presence of model uncertainty (paragraph 1(f)). The starting point could be forecasts for inflation, GDP and the 3M PRIBOR.[40] Naturally, the time series models would have to be regularly tested and re-estimated to be up-to-date. Over time new time series models may need to be developed, depending on the experience with existing models. However, such steps are relatively low-cost for time series models.

Recommendation 2. *As a part of forecast risk assessment, there should be more scope in the forecast process for challenge to the projections and the accompanying narrative.*

FPAS is an advanced and efficient system. Many laborious tasks have been automated over the years and the models comprising FPAS, including the g3+ model, are well integrated with internal databases (DPSZ). The routine operation of g3+ and its satellites seems to be sufficiently supported by the existing IT infrastructure (both software and hardware). A new Data Science Lab has been set up to handle large datasets. The steps of the entire process, including the forecast outputs, are well documented and archived on a shared drive. Based on my observations, the entire process is run in a highly professional manner under tight deadlines. The structure of the process, from the initial Issues Meeting to the Open Monetary Policy Meeting has contours I am familiar with from the Bank of England. The last two meetings between the staff and the Board roughly correspond to the pre-MPC meeting at the Bank of England, comprising both an update on the current conjuncture (accompanied with a comprehensive chartpack) and the forecast. I understand the process is regularly reviewed by the MD management, with the aim to seek further efficiency gains.

The forecast is owned by the staff, whereas the monetary policy decisions are made by the Board on the basis of the forecast and the Board's own perception of risks to the forecast and the economy. Based on the meetings I attended, I have the impression the staff forecast is understood to be a *consensus* forecast of the MD. Once the final version of the forecast is prepared, an advisor to one of the Board members writes up his/her opinion on the forecast. The way I understand the process, this assessment is the only formal challenge to the forecast (a second set of comments, to address financial stability considerations, is written by the Financial Stability Department).

[40] Of course, time series models do not explicitly model the monetary policy reaction function. However, in periods when the economy is not experiencing unusual shocks and monetary policy has not changed its reaction function, time series models (eg, VARs) reflect the average effects of monetary policy on the economy, as well as the endogenous responses of monetary policy to the economy. Indeed, under certain conditions, DSGE models have a VAR representation.

In my view, the challenge to the forecast comes too late and its delivery, in the form of a written note, is a bit too sterile. My recommendation is to bring the challenge forward, change its format and involve the MD staff, in addition to Board advisors. One potential stage at which the forecast challenge could take place is the second meeting with the Board in the forecast process. At that point the first version of the forecast had been prepared and is ready to be debated. One member of the MD and one Board advisor could present, in the presence of the Board and MD staff, their personal perspectives on the economy, the forecast and the accompanying narrative, followed by a discussion. Such a meeting could facilitate a livelier discussion of the current conjuncture and risks to the projections then the existing setup. The forecast would remain squarely in the ownership of the staff but, this way, the risks would be spelled out and debated more clearly in the open. This change may be accompanied with the removal of some duplicate meetings from the MD's internal schedule, in order not to increase the total amount of time the staff spend in meetings. If taken up, this recommendation could be made operational within **one to two years**. In the longer term, it can be tied with the relevant parts of Recommendation 12.

Recommendation 3. *A sustainable long-term process should be put in place for modifications of g3+ and its satellites, to capture alternative transmission mechanisms and to address new policy challenges.*

The g3+ model is a model of effectively only one transmission mechanism of monetary policy (paragraphs 2(m) and 3(d)). Nonetheless, all economic models, however good, represent simplification of reality. It is not difficult, therefore, to come up with a wish list of channels to incorporate into any given model. More often than not, macroeconomic models at central banks get larger over time, as different requests by policymakers are incorporated into the model on the run, often under a tight schedule of forecast rounds. At some point, the model becomes unmanageable and impossible to understand and is scrapped (eg, the BEQM model of the Bank of England). To avoid such perils of macroeconomic modeling at central banks, the CNB should establish a sustainable long-term process that is going to guide the staff and the Board in terms of what potential features are worth explicitly incorporating into g3+, which can be captured by 'quick fixes' in the form of the existing shocks (residuals), and which can be safely left out.

A possible way to implement this process is to step down to a version of g3+ at the level of Smets and Wouters (2007), and perhaps even simpler, and try to understand both the qualitative and ballpark quantitative effects of the additional features in the simpler setup. Based on the findings, a decision can be made about whether the marginal benefits of having the new feature in g3+ outweigh the costs of human resources involved in modifying the model. Examples of new features, on the basis of my discussions with the Board and work streams at other central banks, which could be explored this way, include the effects of firm credit in euro-denominated loans on the transmission mechanism, the mortgage channel of monetary policy and different types of expectations formation.[41] It is ultimately up to the Board and the staff to agree on the work streams.

[41] I am aware that some work on expectations formation and de-anchored inflation expectations has already taken place in the MD and I would only encourage that this work stream continues.

The simpler models could be used also as robustness checks across forecast rounds on g3+ projections and the accompanying economic narrative. Simpler models operate at a higher level of abstraction and thus are unaffected by the details of the data g3+ is required to match. They may, therefore, be less sensitive to data revisions.

Whether this process can be implemented in **one to two years** or **three to four years** depends on the extent to which existing human resources can be deployed for this task (this recommendation can be tied with the relevant parts of Recommendation 12). To make this recommendation operational, the MD would need to get also more support for meeting their IT needs. For instance, at present the usage of Matlab licenses runs at full capacity and any additional use outside of the routine operation of g3+ is difficult. In my experience, such constraints at central banks are unusual.

Recommendation 4. *Some of the conjunctural and due diligence analysis during forecast rounds could be deemphasized and more resources allocated to addressing long-term questions related to monetary policy strategy and risks to the economy not captured by g3+.*

Based on the meetings I attended, and the briefing materials I had access to, the forecast rounds seem to be dominated by the assessment of the current conjuncture, the requirements of the FPAS architecture and explanations of the latest, often idiosyncratic, movements in the data (I was told the forecast round I observed was quite representative). It is my understanding there is also a fair amount of background work, especially in the Monetary Policy and Fiscal Analyses Division and the External Economic Relations Division. Parts of the background work are dictated by the CNB's obligations to external bodies. In comparison with the central banks I have experience with, at the CNB long-term analysis and higher-level policy questions seem to take more of a back seat.

For instance, the natural rate of interest has been a part of practical central banking since the time central banks adopted the NK paradigm as a unifying framework for policy analysis. Indeed, questions regarding the natural rate formed a part of the scenario analyses during the round I observed. Ideally, the CNB would have a few smaller DSGE-NK models (in the spirit of Recommendation 3), in which the implications of the changes in the natural rate could be studied, depending on the underlying potential causes of the change (shocks to technology, changes in longevity and population growth, economic uncertainty, to name a few). Different causes may have different implications for monetary policy. While, due to its complexity, such analysis would be too impractical to carry out in g3+, smaller models can provide economic insights, ballpark estimates and robustness checks to help assess the risks to the forecast.[42]

In a similar spirit, small structural models can be applied to questions regarding optimal monetary policy (see the comments under Recommendation 6), monetary policy communication and fiscal policy considerations. While such work streams may not necessarily address

[42] I am familiar with the existing three approaches used at the CNB to estimate the natural rate. However, they are not structural enough to tackle questions regarding the implications of the causes of the movements in the natural rate.

conjunctural questions of individual forecast rounds, they help policymakers formulate the intellectual basis for the overall monetary policy strategy and its robustness.

There are various risks to the economy not easily incorporated into g3+ and DSGE-NK models. For instance, according to the company's website, ŠKODA AUTO contributes around 5% to Czech GDP. Further, Czechinvest reports that the automobile industry accounts for more than 9% of GDP, 26% of manufacturing and 24% of exports. Models of production networks (supply chains) and economic granularity can be used to study the macroeconomic risks of such exposures and sufficient attention should be paid to these risks and the models. Other risks that will likely impact on monetary policy over the next decade include ageing population and gains (or the lack of) in productivity. The Bank should also have long-term work streams on the interactions between monetary and macroprudential policies. While various risks can be incorporated into DSGE-NK models in reduced-form ways (through various shocks), their deeper analysis often needs to be carried out outside of this paradigm, using different analytical tools.

I suggest the Bank seeks ways to reduce, where possible, the routine briefing and reporting tasks of the MD staff and re-assess the need for various add-ons of the g3+ model (eg, some of the add-ons in the fiscal and external blocks; paragraph 3(g)). The freed-up resources could be allocated to higher-level policy-relevant questions. As in the case of Recommendation 3, whether this suggestions can be implemented in **one to two years** or **three to four years** depends on the extent to which existing human resources can be made available for this task (this recommendation can be tied with the relevant parts of Recommendation 12). The same comments regarding the IT infrastructure as in Recommendation 3 apply here as well.

Recommendation 5. *During forecast rounds, the shocks (residuals) in g3+ could be used as a guidance for further analysis and for building a narrative around the projections.*

One of the advantages of DSGE models is that the models facilitate a decomposition of the observed data in terms of shocks (residuals) with economic interpretation (paragraphs 2(o) and 2(q)). At the same time, reasoning in terms of causality from one endogenous variable to another is invalid in DSGE models (paragraph 2(l)). Based on the briefing materials I had access to and the meetings I attended, most of the narrative around the integrated forecast is built, however, around causalities between the model's endogenous variables. For instance, the narrative for GDP growth is based on consumption growth, which in turn is explained by real wage growth. Such narrative is not informative about the underlying forces at play. One of the points of having a DSGE model is to go beyond such a simple reasoning and uncover the underlying sources of the co-movements among macroeconomic variables.

It would be more informative to build the narrative around the model's shocks. While the shocks cannot be interpreted too narrowly and literally (paragraph 2(q)), they provide leads for further investigation. For instance, during the inflation period 2021-23, the g3+ model required substantial movements in markups to explain observed inflation (a chart on slide 8 of the Meeting with Analysts presentation, 3 May 2024, showing a decomposition into costs and markups; this chart is informative but unfortunately is included neither in the internal nor the published version of the Monetary Policy Report). From the conversations with the staff, my understanding is that a bulk of the movements in the model markups is explained by 'markup shocks'. Looking at the data through the lens of the model, the model is thus suggesting that there was some truth in the story run by the media that firms were taking advantage of their market power to increase their markups during 2021-23 (something not captured by the internal mechanism of the model and therefore picked up by the markup shock). In this respect, the model provided a lead during the forecast rounds for further analysis and a structural narrative for a chunk of the increase in inflation.

There are certain parts of the briefing materials that are informative about the underlying structural forces. Charts 7 and 8 in Section III of the Spring 2004 Monetary Policy Report, which appear in Monetary Policy Reports on a regular basis, provide a decomposition of inflation in terms of the elements of the price tree in g3+ (paragraph 3(d)). They are informative because they have an exact mapping into the monetary transmission mechanism in the model (paragraphs 2(m) and 3(d)). For most parts of the briefing materials, however, the power of the model remains underused. There are no shock decompositions, and thus narratives about underlying economic forces, for GDP and other key data.

My understanding is that within the forecast team, the conversation about the current conjunction and the forecast is in terms of the shock decomposition in g3+. However, most of the time such analysis does not reach the Board. Partly it is due to the challenges in communicating the narrative in terms of the model's shocks (paragraph 2(l)). Naturally, the presentation to the Board cannot be in terms of the narrow technical interpretation of the shocks (habit shocks, adjustment shocks, etc.) but rather in terms of a narrative that can be built around their broader interpretation, ideally supported by additional evidence obtained outside of the model. However, the Board needs to be also open to such a narrative. I was told by contacts at the Riksbank and Norges Bank, users of DSGE-NK models for forecasting and policy analysis, that communication with monetary policymakers at their institutions in terms of the shocks is common. But I am aware of the challenges from my own experience at the Bank of England. If there is an agreement between the Board and the staff on this recommendation, it could be implemented almost immediately and certainly within **one to two years**.

Recommendation 6. *The normative interpretation of the current monetary policy rule in g3+ needs to be deemphasized.*

The monetary policy rule in the g3+ model specifies the short-term nominal interest rate as responding to expected inflation (with some interest rate smoothing). The policy rule is typically interpreted in a normative way, as a rule ensuring the achievement of the CNB's inflation target at the monetary policy horizon. This interpretation should be deemphasized. Unless other parameters of the model, and especially the persistence parameters of the shocks, are just right, the monetary policy rule does not guarantee this outcome in the model (see the Technical appendix for details). A more appropriate interpretation would be the interpretation adopted by the Riksbank "to capture the historical behavior of the Riksbank"; see page 30 of Adolfson, Laséen, Christiano, Trabandt, and Walentin (2013). The role of the rule is simply to give the agents in the model guidance for forming expectations on the basis of the typical responses of monetary policy to the economy. On this ground, the DSGE-NK model of the Riksbank (and also of the Bank of England) specifies the policy rule as responding also to measures of economic activity.[43] It is indeed possible for a policy rule to respond to variables other than (expected) inflation and not violate the inflation targeting objective (Woodford, 2003). The Norges Bank's NEMO, for instance, allows for a derivation of the optimal policy rule from the minimization of the the central bank's loss function, in which case the policy rule responds also to the shocks hitting the economy.[44] Notwithstanding to say that the above central banks, like the CNB, are inflation targeting central banks. I would recommend that in the next **one to two years** the CNB replaces the normative interpretation of the current policy rule in g3+ with a positive interpretation along the lines of the Riksbank. Such an interpretation would facilitate also easier communication of any potential deviations of the Board's decisions from a path implied by the rule (paragraph 1(f)). In the longer run, over the next **three to four years**, the Bank could consider engaging in the monetary strategy work streams proposed under Recommendation 4.

Recommendation 7. *The UIP component of the transmission mechanism should be deemphasized and possibly removed.*

In line with no-arbitrage principles, the exchange rate in the g3+ model is determined by the no-arbitrage condition introduced in paragraph 2(m), with the rate of return given by the domestic currency return on a foreign bond financed by a domestic bond. As in most other DSGE-NK models, the no-arbitrage condition is implemented in its linear form (paragraph 2(n)), resulting in the standard uncovered interest rate parity (UIP) condition. A largely exogenous risk premium and ad-hoc dynamics are then attached to the UIP condition (paragraph 3(d)). The exchange rate in the model is thus partially determined endogenously through UIP (and the ad-hoc dynamics) and partially exogenously through an appropriate parameterization of the risk premium. Since Fama (1984), a large literature has documented a systematic failure of UIP in the data. Another strand of the literature, starting with Meese and Rogoff (1983), shows that exchange rates are reasonably well approximated by a random walk; in which case, the interest rate differential in the no-arbitrage condition corresponds roughly to the risk premium. It would therefore be advisable to deemphasize the UIP component of the exchange rate determination in g3+ and possibly model the exchange rate simply as an exogenous random walk process (possibly with a drift to reflect long-run convergence of the Czech economy to the EU average). If resources allow, the Bank could start a work stream on using high-frequency identification (Kuttner, 2001) and local projection methods to better understand the effects of monetary policy on the exchange rate and its propagation to the economy. These recommendations could be taken up within **one to two years**.

[43] See Burgess, Fernandez-Corugedo, Groth, Harrison, Monti, Theodoridis, and Waldron (2013) and Adolfson et al. (2013).

[44] Brubakk, Anders Husebo, Maih, Olsen, and Ostnor (2006).

Recommendation 8. *In periods of extreme events, it needs to recognized that the structural parameters of g3+ are not strictly invariant. In such circumstances, the relevant parameters should be recalibrated.*

Model parameters are structural only in the context of their model abstraction. In g3+, the Calvo and indexation parameters, for instance, are not structural outside of the model. These parameters control nominal price rigidities, and thus the steepness of the Phillips curve, in the model. The nominal price stickiness implied by the Calvo and indexation parameters is not invariant, however, in the context of state-dependent models of price setting. In such models, the fraction of firms adjusting prices is not constant but varies with the state of the economy (eg, the fraction is larger in periods of high inflation). Thus, while the Calvo and indexation parameters may be a reasonable approximation of reality in times of normal-size shocks, they become an unreliable guidance in times of extreme events. This situation may have occurred during the forecast rounds in 2021 (Appendix to the Spring 2023 Monetary Policy Report, Charts 2 and 3). For 2021 and the start of 2022 the CNB was forecasting a relatively mild increase in inflation accompanied by subdued real GDP growth. In reality, inflation was much higher and the real economy more robust (ie, the Phillips curve was steeper than implied by the forecast). I understand the price stickiness parameters in g3+ were kept constant at their usual values during this period. Even if this aspect of the model was not responsible for the entire forecast error, it would be prudent to reassess the structural nature of the model parameters in times of extreme events and adjust the values of the relevant parameters accordingly. While there may be some technical issues involved, it should be possible to implement the recommendation almost immediately, certainly within **one to two years**.

Recommendation 9. *The documentation of g3+ and its satellites needs to be improved and non-critical features removed from the model.*

The most detailed description of the g3+ model is provided by two CNB working papers, Andrle et al. (2009) and Brázdik et al. (2020). In addition, the Winter 2024 Monetary Policy Report contains a brief description of the most recent updates. Specific aspects of the model are described in CNB blogs and various Monetary Policy Reports. In addition, the general features of the model and FPAS are introduced in internal documents distributed to new Board members. The external reviewers were also provided with a document describing both g3+ and FPAS. Unfortunately, I did not find any of these documents sufficiently informative about the details of g3+ and its integration into FPAS. For instance, the two working papers seem to attempt to strike a balance between being overly technical and providing detailed description of the model. In my opinion, they do not achieve either goal. For the casual reader, they are too involved, while for an economist with PhD-level training in macroeconomics they are not detailed enough (eg, the parameterization methodology is too vaguely described, a comprehensive documentation of the parameter values is missing, a complete list of the equilibrium conditions is not provided, the forcing processes are not clearly specified, the description of the fiscal block is outdated). The missing details, nonetheless, are critical for comprehensive understanding of the model's properties and its outcomes. The same observations apply even more to the satellite models, whose description is even more sparse. In the end, I obtained the missing information through in-person meetings and e-mail exchanges with the members of the forecast team most closely involved in the development and operation of the model. But this information should be easily accessible.

The g3+ model and its satellites need to be precisely documented. All relationships within the model, and between the model and its satellites, need to be precisely documented in a mathematical form. This is important not only for internal and external scrutiny but also for business continuity. The publicly available descriptions of the main DSGE-NK models used at the Bank of England, Riksbank, Bank of Finland, and the Norges Bank, while not perfect, are substantially more satisfactory.[45] The descriptions of COMPASS (the Bank of England) and Ramses II (Riksbank), in particular, are close to what one would expect. It should be possible to implement this recommendation within **one to two years**.

During the documentation process, the staff should also take stock of the shocks in the model and reassess the way the fiscal impulse is modeled. It appears various shocks were added to the model over the years on the run and their identification and role are not clear. Modeling of the fiscal impulse relies on a combination of the fiscal add-in and ad-hoc shocks to preferences and adjustment costs (paragraph 3(f)). The combination of external multipliers in the fiscal add-in and shocks that have nothing to do with fiscal policy makes the value added of g3+ for analyzing the effects of fiscal policy dubious. Given that g3+ has hand-to-mouth households, too strong consumption smoothing that perhaps motivated the current modeling of the fiscal impulse should be less of an issue in g3+ than in g3. And rather than using external multipliers, an empirically plausible fiscal multipliers could be generated endogenously by the expected persistence of the fiscal measures. The proposed clean-up of the model, especially of the fiscal block, would likely take **three to four years**.

Recommendation 10. *The presentation of the forecast should clearly indicate which projections come from g3+ and which come from its satellites or near-term-forecast models.*

The CNB forecast is an integrated forecast, reflecting not only the structure of g3+, but also judgement, satellite add-ins and satellite add-ons (paragraphs 3(e)-(h)). Some of the projections, including their decompositions, come directly from g3+ (conditional on the inputs from judgement and satellite add-ins), while others are decompositions of the g3+ projections based on add-ons. For instance, some inflation decompositions come directly from g3+, while others come from add-ons. In addition, various forecasts from NTF models are also presented in the briefing documents and presentations. Further, in some cases (eg the foreign block) there also add-on decompositions of the add-ins. For those not familiar with all the details of FPAS it is difficult to always understand the sources of the projections. The briefing documents and presentations would be more transparent, and thus more open to internal scrutiny and challenge, if each chart clearly indicated what models the projections in the chart are based on. For instance, the projections could be labeled as 'g3+ integrated forecast', 'add-on decomposition of the g3+ integrated forecast', 'NTF forecast', etc. This recommendation, if taken up, can be implemented fairly quickly, certainly within **one to two years**.

[45] See, respectively, Burgess et al. (2013), Adolfson et al. (2013), Silvo and Verona (2020), and Brubakk et al. (2006)

Recommendation 11. *The CNB needs to ensure it keeps the technical know-how and human capital to operate, maintain and develop its analytical tools.*

The g3+ model is by far the most complex part of FPAS. I was told it takes about one and a half year for a newcomer to learn how to operate the model and about five years to understand the model in enough detail to modify it in a nontrivial way. As things are, a departure of two to three economists most familiar with the model, and having the required technical skills, would make any major modifications and extensions of the model impossible. The human resources strategy of the CNB needs to recognize this problem and ensure the CNB remains an attractive work place for highly qualified staff. At the same time, the CNB needs to ensure business continuity is preserved in the case of the critical staff departures. It is beyond the scope of my remit to make concrete suggestions in this direction. I imagine that some short-term measures could be introduced over the next **one to two years**, while developing and expanding the necessary human capital needed for business continuity will likely take **three to four years**.

Recommendation 12. *The CNB needs to invest in economic research to be better prepared for new challenges, to keep up-to-date with the relevant techniques and literatures and maintain healthy internal exchange of alternative views and ideas.*

Research at the CNB has a long tradition but has never been at its full potential and in the past decade has been deemphasized. This is in stark contrast to the trends at other central banks. While some central banks[46] have always placed heavy emphasis on research, other central banks have made conscious effort in the past 10 to 20 years to build up their research capacities.[47] If the situation at the CNB does not improve, in ten years the Bank will find itself being stuck in an institutional groupthink and intellectual isolation from its peers. It will be impossible to keep up with the latest advances in research, modeling techniques and discourse among central banks.

Forecast and briefing rounds at central banks operate under very tight schedules. In such an environment, it is easy to stop seeing the forest for the trees and for groupthink to develop. In conversations with research directors at different central banks, and current or former interest rate setting committee members, the ability to challenge, provide a fresh perspective and think outside of the box are mentioned as the key attributes of a strong research unit. In addition, research staff provide technical assistance with model development, disseminate within the organization the latest academic research, bring in international experts through their networks, assist with the preparation of policymakers' speeches, and increase the institution's intellectual firepower to push various agendas at international forums.

For all the above reasons, the CNB needs to invest significantly more in economic research. Not only by providing sufficient resources to the existing research units, but also by having

[46] The Board of Governors, all regional banks of the Federal Reserve System, ECB, and the Bundesbank.

[47] Among others, the Bank of Canada, Danmarks Nationalbank, Bank of England, Banco de España, Bank of Finland, Banque de France, Banca d'Italia, Bank of Lithuania, De Nederlandsche Bank, Norges Bank, Banco de Portugal, Riksbank, and recently also Národná Banka Slovenska.

a sustainable long-term plan for expanding its research capacity.[48] Looking at the combined numbers for the Monetary and Financial Stability Departments, research staff make up only 0.67% of the CNB's employees. This compares with 1% at the RBA, 2.1% at the Danmarks Nationalbank, 3.1% at the Bank of Finland, 2.2% at the Bank of Lithuania, 1.3% at De Nederlandsche Bank, 1.2% at the Norges Bank, and 3.1% at the Riksbank (these are the banks for which I have the data, not counting research assistants and administrative staff; regional Feds, in which research plays a much stronger role than at other central banks, are excluded).

The constraints for the existing research staff, the way I understand the situation, range from insufficient number of Matlab (and other specialized software) licenses to budget cuts for organizing seminars and conferences, which are important to keep the research staff in contact with international research networks and are common at other central banks. I recommend that these relatively small IT and budgetary problems are addressed within the next **one to two years**.

To build up a stronger and larger research unit is, however, a much more challenging task. The parameters of the unit need to be right to ensure its long-term potential at the Bank. The CNB should seek to make substantial progress in this direction in the next **three to four years**. Given the size of the Bank, ultimately doubling the staff numbers in the research unit is not unreasonable. However, there are a number of aspects of this process, listed below, to be aware of and the Bank should take up my recommendation only if there is sufficiently strong long-term commitment and support for this recommendation among the Board members.

First, the research unit has to have research staff interested not only in research but also in policy work. Communication skills are equally important. The age composition should comprise both senior members, who get relatively more involved in policy work, and junior staff, who bring in the latest tools and techniques from their PhD training and get relatively more time for research. There should be analytical diversity, spanning areas from DSGE macroeconomics and time series econometrics to finance, labor, and micro-level econometrics.

Second, the research unit needs to contribute to policy work in some of the ways mentioned above. It could also take up some of the tasks mentioned in Recommendations 2-4 (forecast challenge, assistance with model development, addressing long-term policy questions).

Third, the time allocation of the research staff needs to allow for independent undirected research, not necessarily on the Czech economy. At the central banks mentioned in the first paragraph of this Recommendation, 50% is the minimum and in the case of junior staff (ie, fresh from a PhD programme) the share is even larger. Independent undirected research keeps the staff in connection with the research frontier, with the resulting publications providing an external validation of their expertise. Long-term experience of other central banks shows that such a work arrangement is necessary to attract the talent required for research

[48]Currently, the CNB has two research units, one in the MD and another in the Financial Stability Department. Although, as per my remit, my focus is on the MD, some of the comments are of general nature.

units to play their role in the policy process.[49] To ensure the undirected research is of sufficiently high quality by international standards, central banks typically operate points systems based on publications, weighted by journal rankings. Those who systematically underperform are assigned to more policy-oriented roles.

Fourth, the CNB will have to commit sufficient resources for external research seminars and presentations of its research staff at international conferences. The frequency of external seminars and attendance of conference should be comparable to other central banks. The Bank will also need to be flexible with IT and specialized software requirements.

Finally, the CNB, like other central banks, will have to look for research staff not only locally but also outside of the national borders. The need to be outward looking is especially pertinent for small countries in which the education system is dominated by a few universities and thus the danger of a long-term groupthink is more real than in larger economies. Realistically, the CNB should aim to compete for research staff with some of the smaller national central banks and second-tier European universities. There are substantial search costs involved in the process (mainly staff's time and to a lesser extent also monetary) and the HR Department and other operational units will have to be flexible to work with foreign nationals.

Recommendation 13. *The CNB should continue investing in its databases. However, some areas of its data management need to be reorganized to free up the time of the analytical staff.*

The CNB has made great progress in the scope of its databases (DPSZ), their automatization, management, and integration with FPAS and g3+. Key data are automatically uploaded into FPAS and standard charts, including those in the chartpack and slides for meetings with the Board, are produced automatically.

The CNB has also significantly improved the interface and functionality of its publicly available ARAD database. The interface and functionality of ARAD are currently comparable to the interface and functionality of the FRED database, possibly the most widely used free database on the US and international macroeconomic data. The CNB should continue to monitor the trends in database functionality at other institutions to keep the functionality of ARAD up-to-date. Importantly, the organization of ARAD should continue to preserve its sectoral tree structure and the structure of national accounts (as currently implemented under the 'Indicators' tab in the main menu). A switch to a purely search-engine-style database, which has been adopted by some institutions, would be highly counterproductive.

In recent years, the NTF team have acquired a number of firm- and household-level databases and engaged in webscraping. These data are used for backcasting, nowcasting and near-term forecasts. The management of these databases, as well as webscraping, rests with the NTF team. This arrangement, a relict of the team's original initiative in these directions, takes up significant amount of the NTF team's time (only the interface is managed by a dedicated

[49] Such a setup has been recently endorsed by the Bernanke (2024) Review of the Bank of England, which has only recently moved from fully directed to 50%+ independent undirected research.

IT developer, whose assignment to the team is on a part-time basis). While some continued involvement of the NTF team in the management of the databases would be desirable, most of the actual work should be ideally off-loaded to a specialized data management unit. This would free up the time of the economists in the NTF team to carry out more analysis on the data, which at present remain rather under-explored. It should be possible to implement such a change within **one to two years**. Any changes to the subscriptions to the micro-level databases should be consulted with the NTF team and the MD at large. The staff have expressed concerns that the subscriptions may be discontinued by the budget holders outside of the MD, without sufficient appreciation for business continuity of the databases in FPAS and economic research.

References

Adolfson, M., Laséen, S., Christiano, L., Trabandt, M., Walentin, K., 2013. Ramses II - model description. Occasional Paper Series 12/2013, Sveriges Riksbank.

Andolfado, D., 1996. Business cycle and labor-market search. American Economic Review 86, 112–32.

Andrle, M., Hlédik, T., Kameník, O., Vlček, J., 2009. Implementing the new structural model of the Czech National Bank. Working Paper Series 2/2009, Czech National Bank.

Bank of England, 1999. The economic models at the Bank of England. London.

Banque de France, 2019. The FR-BDF model and an assessment of monetary policy transmission in France. Working Paper Series, No. 736.

Benecká, S., Kábrt, M., Komárek, L., 2024. Central banks macroeconomic forecasts: When two do the same thing, it is not the same thing. Global economic outlook, 8 march 2024, Czech National Bank.

Bernanke, B. S., 2024. Forecasting for Monetary Policy Making and Communication at the Bank of England: A Review. Bank of England.

Beyer, A., Reichlin, L., 2008. The Role of Money - Money and Monetary Policy in the Twenty-First Century. European Central Bank.

Bobeica, E., Hartwig, B., 2023. The COVID-19 shock and challenges for inflation modelling. International Journal of Forecasting 39, 519–539.

Brayton, F., Levin, A., Tryon, R., Williams, J. C., 1997. The evolution of macro models at the Federal Reserve Board. Tech. rep., Federal Reserve Board.

Brázdik, F., Hlédik, T., Humplová, Z., Martonosi, I., Musil, K., Ryšánek, J., Šestořád, T., Tonner, J., Tvrz, S., Žáček, J., 2020. The g3+ model: An upgrade of the Czech National Bank's core forecasting framework. Working Paper Series 7/2020, Czech National Bank.

Brubakk, L., Anders Husebo, T., Maih, J., Olsen, K., Ostnor, M., 2006. Finding NEMO: Documentation of the norwegian economy model. Staff Memo 6, Norges Bank.

Burgess, S., Fernandez-Corugedo, E., Groth, C., Harrison, R., Monti, F., Theodoridis, K., Waldron, M., 2013. The Bank of England's forecasting platform: COMPASS, MAPS, EASE and the suite of models. Working Paper 471, Bank of England.

Chari, V. V., Kehoe, P. J., McGrattan, E. R., 2009. New-Keynesian models not yet useful for monetary policy analysis. American Economic Journal: Macroeconomics 1, 242–66.

Christiano, L. J., Eichenbaum, M., Evans, C. L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1–45.

Cobham, D., 2003. The Making of Monetary Policy in the UK, 1975-2000. John Wiley & Sons, New York.

Cochrane, J., 2011. Determinacy and identification with Taylor rules. Journal of Political Economy 119, 565–615.

Constâncio, V., 2017. Developing models for policy analysis in central banks. Opening speech at the Annual Research Conference, Frankfurt am Main, 25-26 September 2017.

Del Negro, M., Dogra, K., Gleich, A., Gundam, P., Lee, D., Nallamotu, R., Pacula, B., 2024. The NY Fed DSGE Model: A post-COVID assessment. AEA Papers and Proceedings 114, 95–100.

Duesenberry, J. S., Fromm, G., Klein, L. R., Kuh, E., 1969. The Brookings Model: Some Further Results. North-Holland Publishing Company, Amsterdam.

European Central Bank, 2021. Review of macroeconomic modelling in the Eurosystem: Current practices and scope for improvement. Occasional Paper Series, No. 267.

European Central Bank, 2024. ECB macroeconometric models for forecasting and policy analysis. Occasional Paper Series, No. 344.

Evans, M. K., 1969. Macroeconomic Activity: Theory, Forecasting, and Control. Harper & Row Publishers, New York.

Fair, R. C., 1984. Specification, Estimation, and Analysis of Macroeconomic Models. Harvard University Press, Cambridge, Massachusetts.

Fama, E. F., 1984. Forward and spot exchange rates. Journal of Monetary Economics 14, 19–38.

Favero, C. A., 2001. Applied Macroeconomics. Oxford University Press, Oxford.

Federal Reserve Board, 1996. A guide to FRB/US: A macroeconomic model of the United States. Washington, D.C.

Galí, J., 2015. Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications, 3rd Edition. Princeton University Press, Princeton.

Gosselin, M., Kozicki, S., 2023. Making it real: Bringing research models into central bank projections. Staff Discussion Paper 2023-29, Bank of Canada.

Haertel, T., Hamburg, B., Kusin, V., 2022. The macroeconometric model of the Bundesbank revisited. Technical paper 01/2022, Deutsche Bundesbank.

Kuttner, K. N., 2001. Monetary policy surprises and interest rates: Evidence from the fed funds futures market. Journal of Monetary Economics 47, 523–44.

Meese, R. A., Rogoff, K., 1983. Empirical exchange rate models of the seventies: Do they fit out of sample? Journal of International Economics 14, 3–24.

Silvo, A., Verona, F., 2020. The Aino 3.0 model. Discussion Paper 9, Bank of Finland.

Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro area. Journal of the European Economic Association 1, 1123–75.

Smets, F., Wouters, R., 2007. Shocks and frictions in U.S. business cycles: A Bayesian DSGE approach. American Economic Review 97, 586–606.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

Annex: Technical appendix

The policy rule and the convergence of inflation to target

The model used in this appendix is a stylized DSGE-NK model, which can be thought of as a severely stripped-down version of g3+. Due to its simple structure, the main result can be shown analytically. The fact that g3+ is significantly more complex does not invalidate the point made here. If anything, the rich structure of g3+, with many exogenous variables and shocks, makes the point stronger.

As in g3+ the period utility function is separable in consumption and labor and contains external habits in consumption (see Andrle et al., 2009)

$$u = \frac{(c_t - \tau\psi_t C_{t-1})^{1-\alpha}}{1-\alpha} - \frac{n^{1+\eta}}{1+\eta} \quad \alpha, \tau, \eta \geq 0.$$

Here, c_t is individual consumption, C_{t-1} is aggregate consumption in the previous period, n_t is individual labor, and ψ_t is exogenous stochastic variation in the weight on past consumption. The representative household maximizes the standard time-additive expected utility subject to state-dependent sequences of period budget constraints. The production function is assumed to depend only on labor

$$y_t = an_t,$$

where a is a constant productivity level. The monetary policy rule is the same as in $g3+$, except that it depends on expected inflation one period ahead, instead of four periods ahead (Andrle et al., 2009). This assumption simplifies the algebra without invalidating the message. Specifically, the monetary policy rule is

$$i_t = i + (1-\phi)\nu_\pi E_t\pi_{t+1} + \phi(i_{t-1} - i) + \xi_t, \quad \phi \in [0,1), \nu_\pi > 1,$$

where i_t is the nominal interest rate, i is the natural rate, π_{t+1} is the deviation of the inflation rate from target one period ahead, and ξ_t is a monetary policy shock.[50] Firms face Calvo-style nominal price rigidities with markup shocks. As in g3+ firms that in a given period do not optimally adjust prices resort to backward indexation.

In its log-linear form, the model consists of a linear Euler equation (IS curve), the NK Phillips curve and the monetary policy rule, respectively,

$$\hat{y}_t = h_0\hat{y}_{t-1} + h_1 E_t\hat{y}_{t+1} - h_2(\hat{i}_t - E_t\pi_{t+1}) + \mu_t, \tag{1}$$

$$\pi_t = g_0\hat{y}_t - g_1\hat{y}_{t-1} + \beta^* E_t\pi_{t+1} + \gamma\pi_{t-1} + \zeta_t, \tag{2}$$

$$\hat{i}_t = m_0 E_t\pi_{t+1} + \phi\hat{i}_{t-1} + \xi_t. \tag{3}$$

Here, $\hat{y}_t$ is the percentage deviation of output from steady state and $\hat{i}_t$ is the percentage point deviation of the nominal interest rate from the natural rate. Further,

$$h_0 \equiv \frac{\tau}{1+\tau}, \quad h_1 \equiv \frac{1}{1+\tau}, \quad h_2 \equiv \frac{1-\tau}{\alpha(1+\tau)},$$

[50] As in Andrle et al. (2009) the natural rate of interest is constant.

$$g_0 \equiv \Omega^* \left(\frac{\alpha}{1-\tau} + \eta \right), \quad g_1 \equiv \Omega^* \frac{\tau\alpha}{1-\tau}, \quad m_0 \equiv (1-\phi)\nu_\pi,$$

and $\Omega^* \equiv \Omega/(1+\omega\beta)$, $\Omega \equiv (1-\theta\beta)(1-\theta)/\theta$, $\beta^* \equiv \beta/(1+\omega\beta)$, $\gamma \equiv \omega/(1+\omega\beta)$, and $\omega \in [0,1]$ is the indexation parameter in the firms' problem, $\beta \in (0,1)$ is the time preference parameter of the representative household and $\theta \in [0,1]$ is the Calvo parameter. Finally, the shocks μ_t, ζ_t, ξ_t follow AR(1) processes

$$\mu_{t+1} = \rho_\mu \mu_t + \epsilon^\mu_{t+1},$$

$$\zeta_{t+1} = \rho_\zeta \zeta_t + \epsilon^\zeta_{t+1},$$

$$\xi_{t+1} = \rho_\xi \xi_t + \epsilon^\xi_{t+1},$$

where ϵ^μ_{t+1}, ϵ^ζ_{t+1}, ϵ^ξ_{t+1} are mutually independent innovations, ρ_μ, ρ_ζ, $\rho_\xi \in [0,1)$ and μ_t is a normalized habits shock.

To proceed, I remove μ_t and ξ_t (the habits and monetary policy shocks) for the moment from the model. This reduces the number of equations below. Once I demonstrate the main point for this simplified case, I will generalize to the case of multiple shocks. Under standard determinacy conditions, the system (1)-(3) admits linear solutions of the form

$$\hat{y}_t = a_y \hat{y}_{t-1} + a_\pi \pi_{t-1} + a_i \hat{i}_{t-1} + a_\zeta \zeta_t, \tag{4}$$

$$\pi_t = b_y \hat{y}_{t-1} + b_\pi \pi_{t-1} + b_i \hat{i}_{t-1} + b_\zeta \zeta_t, \tag{5}$$

$$\hat{i}_t = d_y \hat{y}_{t-1} + d_\pi \pi_{t-1} + d_i \hat{i}_{t-1} + d_\zeta \zeta_t, \tag{6}$$

where the coefficients a_y, a_π, a_i, a_ζ, b_y, b_π, b_i, b_ζ, d_y, d_π, d_i, d_ζ are the unknowns to be determined. For analytical purposes, I use the method of undetermined coefficients, which leads to twelve restrictions determining the twelve coefficients. The twelve restrictions have a recursive structure, whereby the solution for a_y, a_π, a_i, b_y, b_π, b_i, d_y, d_π, d_i is independent of the solution for a_ζ, b_ζ, d_ζ.

The nine restrictions for the first set of the nine coefficients a_y, a_π, a_i, b_y, b_π, b_i, d_y, d_π, d_i are

$$h_0 + (h_1 a_\pi + h_2 b_\pi) b_y + (h_1 a_i + h_2 b_i - h_2) d_y - (1 - h_1 a_y - h_2 b_y) a_y = 0, \tag{7}$$

$$(h_1 a_\pi + h_2 b_\pi) b_\pi + (h_1 a_i + h_2 b_i - h_2) d_\pi - (1 - h_1 a_y - h_2 b_y) a_\pi = 0, \tag{8}$$

$$(h_1 a_\pi + h_2 b_\pi) b_i + (h_1 a_i + h_2 b_i - h_2) d_i - (1 - h_1 a_y - h_2 b_y) a_i = 0, \tag{9}$$

$$(g_0 + \beta^* b_y) a_y + \beta^* b_i d_y - g_1 - (1 - \beta^* b_\pi) b_y = 0, \tag{10}$$

$$(g_0 + \beta^* b_y) a_\pi + \beta^* b_i d_\pi + \gamma - (1 - \beta^* b_\pi) b_\pi = 0, \tag{11}$$

$$(g_0 + \beta^* b_y) a_i + \beta^* b_i d_i - (1 - \beta^* b_\pi) b_i = 0, \tag{12}$$

$$m_0 b_y a_y + m_0 b_\pi b_y - (1 - m_0 b_i) d_y = 0, \tag{13}$$

$$m_0 b_y a_\pi + m_0 b_\pi^2 - (1 - m_0 b_i) d_\pi = 0, \tag{14}$$

$$m_0 b_y a_i + m_0 b_\pi b_i + \phi - (1 - m_0 b_i) d_i = 0. \tag{15}$$

Observe that neither a_ζ, b_ζ, d_ζ, nor ρ_ζ, appear in the system (7)-(15). This system determines the nine coefficients loading onto the endogenous state variables $\hat{y}_{t-1}$, π_{t-1}, $\hat{i}_{t-1}$ in the solution (4)-(6). These coefficients are independent of ρ_ζ, the persistence of the markup shock. This result is a consequence of the certainty equivalence property of linear models.

The remaining three restrictions are

$$(h_1 a_\zeta + h_2 b_\zeta)\rho_\zeta + (h_1 a_\pi + h_2 b_\pi)b_\zeta + (h_1 a_i + h_2 b_i - h_2)d_\zeta - (1 - h_1 a_y - h_2 b_y)a_\zeta = 0, \quad (16)$$

$$(g_0 + \beta^* b_y)a_\zeta + \beta^* b_i d_\zeta + (\beta^* b_\zeta \rho_\zeta + 1) - (1 - \beta^* b_\pi)b_\zeta = 0, \quad (17)$$

$$m_0 b_y a_\zeta + m_0 b_\pi b_\zeta + m_0 b_\zeta \rho_\zeta - (1 - m_0 b_i)d_\zeta = 0. \quad (18)$$

The system (16)-(18) determines a_ζ, b_ζ, d_ζ, given the solution for a_y, a_π, a_i, b_y, b_π, b_i, d_y, d_π, d_i. Observe that ρ_ζ appears in (16)-(18). The coefficients in (4)-(6) loading onto the shock ζ_t thus depend on the persistence of the shock.

The entire system (7)-(18) depends, however, on the two parameters of the monetary policy rule, ν_π and ϕ (refer back to the definition of m_0). Thus, all twelve coefficients a_y, a_π, a_i, b_y, b_π, b_i, d_y, d_π, d_i and a_ζ, b_ζ, d_ζ depend, in principle, on ν_π and ϕ.

More generally, in the case with all three shocks, the solution of the model has the form

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \\ \hat{i}_t \end{bmatrix} = A \begin{bmatrix} \hat{y}_{t-1} \\ \pi_{t-1} \\ \hat{i}_{t-1} \end{bmatrix} + \begin{bmatrix} \lambda_\mu & \lambda_\zeta & \lambda_\xi \end{bmatrix} \begin{bmatrix} \mu_t \\ \zeta_t \\ \xi_t \end{bmatrix},$$

where A is a 3-by-3 matrix and λ_μ, λ_ζ, λ_ξ are 3-by-1 vectors. The elements of the matrix and the vectors are nonlinear functions of the model's deep parameters, whereby the elements of A are determined by (7)-(15), the elements of λ_ζ by (16)-(18) and the elements of λ_μ and λ_ξ by another system of three equations each, with the same recursive relationship to (7)-(15) as the system (16)-(18). We can thus write $A(\nu_\pi, \phi)$ and $\lambda_\mu(\nu_\pi, \phi, \rho_\mu)$, $\lambda_\zeta(\nu_\pi, \phi, \rho_\zeta)$, $\lambda_\xi(\nu_\pi, \phi, \rho_\xi)$.

The expected speed of convergence of $\hat{y}_t$, π_t and $\hat{i}_t$ to steady state, and thus the expected speed of convergence of inflation to target and of the interest rate to the natural rate, depends on the internal dynamics of the system, driven by the highest eigenvalue of A, and the persistence of the shocks, determined by the parameters ρ_μ, ρ_ζ and ρ_ξ. The relative importance of the shocks for the speed of convergence depends on λ_μ, λ_ζ and λ_ξ. Monetary policy is characterized by two parameters, ν_π and ϕ. Thus, given all other parameters of the model, and in particular the persistence parameters of the shocks, the two monetary policy parameters alone cannot guarantee convergence of the system to steady state at any particular speed (time horizon). The richer is the model, and thus the more shocks there are in the model, the more difficult it becomes for the policy rule (3) to achieve the desired speed of convergence, unless the persistence of the shocks, and possibly the values of other parameters, are calibrated in a way to achieve this objective. In the current calibration of g3+, many persistence parameters are 'fine tuned' to achieve desirable model properties.